

Similar Shapes - Areas and Volumes

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The lengths of a shape are multiplied by 3 to make a similar shape.

By what number is the area multiplied?

Circle your answer.

Options: 3, 6, 9, 27

Area scale factor = (length scale factor)²

$$3^2 = 9$$

Answer: 9

2 (2 marks)

Two similar triangles have corresponding sides 4 cm and 10 cm.

The area of the smaller triangle is 12 cm².

Work out the area of the larger triangle.

Length scale factor = $10 \div 4 = 2.5$

Area scale factor = $2.5^2 = 6.25$

Area of larger triangle = $12 \times 6.25 = 75$

Answer: 75 cm²

Marks: 1 mark for 12×2.5^2 . 1 mark for 75.

3 (1 mark)

Two rugs are mathematically similar.

The area of the larger rug is 4 times the area of the smaller rug.

Zainab says,

“The larger rug is 4 times as long as the smaller rug.”

Is she correct?

Tick a box.

Give a reason for your answer.

Area scale factor = 4

A length is found by square rooting the area scale factor.

Length scale factor = $\sqrt{4} = 2$

So the larger rug is 2 times as long, not 4 times.

Answer: No. The length scale factor is $\sqrt{4} = 2$, so it is 2 times as long.

Tip: Just ticking “No” is not enough. Give the reason with the number: the length scale factor is $\sqrt{4} = 2$.

4 (1 mark)

A cuboid has volume 15 cm^3 .

It is enlarged by scale factor 2 to make a similar cuboid.

What is the volume of the enlarged cuboid?

Circle your answer.

Options: 30 cm^3 , 60 cm^3 , 120 cm^3 , 240 cm^3

Volume scale factor = (length scale factor)³

$2^3 = 8$

Volume = $15 \times 8 = 120 \text{ cm}^3$

Answer: 120 cm^3

Tip: Doubling every length does not double the volume. The volume is multiplied by $2^3 = 8$.

5 (2 marks)

Each row of the table is for a different pair of similar solids.
Complete the table.

Length scale factor	Area scale factor	Volume scale factor
2	4	8
5	25	125

First row: area scale factor = $2^2 = 4$ and volume scale factor = $2^3 = 8$

Second row: length scale factor = $\sqrt{25} = 5$

Then volume scale factor = $5^3 = 125$

Answer: 4 and 8; 5 and 125

Tip: In the second row, go back to the length scale factor first ($\sqrt{25} = 5$), then cube it.
 $25^2 = 625$ is wrong.

Marks: 1 mark for 4 and 8. 1 mark for 5 and 125.

6 (3 marks)

Two similar cylinders have volumes 54 cm^3 and 250 cm^3 .

The height of the smaller cylinder is 6 cm.

Work out the height of the larger cylinder.

$$\text{Volume scale factor} = \frac{250}{54} = \frac{125}{27}$$

Cube root to get the length scale factor:

$$\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$$

$$\text{Height of larger cylinder} = 6 \times \frac{5}{3} = 10$$

Answer: 10 cm

Marks: 1 mark for $250 \div 54$. 1 mark for taking the cube root. 1 mark for 10.

7 (4 marks)

Two bottles are mathematically similar.

The smaller bottle is 10 cm tall and has surface area 240 cm^2 .

The larger bottle is 15 cm tall.

- (a) Work out the surface area of the larger bottle.

$$\text{Length scale factor} = \frac{15}{10} = 1.5$$

$$\text{Area scale factor} = 1.5^2 = 2.25$$

$$\text{Surface area} = 240 \times 2.25 = 540$$

Answer: 540 cm^2

Marks: 1 mark for 240×1.5^2 . 1 mark for 540.

- (b) The larger bottle holds 1.35 litres.
How much does the smaller bottle hold?

$$\text{Volume scale factor} = 1.5^3 = 3.375$$

The smaller bottle holds less, so divide:

$$1.35 \div 3.375 = 0.4$$

Answer: 0.4 litres

Tip: For volume (capacity) use the cube of the length scale factor, not the area scale factor from part (a).

Marks: 1 mark for $1.35 \div 1.5^3$. 1 mark for 0.4.

8 (3 marks)

A shop sells two sizes of solid chocolate egg.

The eggs are mathematically similar and made of the same chocolate.

The small egg has mass 27 g and is wrapped in 45 cm^2 of foil.

The large egg has mass 216 g.

Work out the area of foil needed to wrap the large egg.

The eggs are made of the same chocolate, so mass is in the same ratio as volume.

$$\text{Volume scale factor} = 216 \div 27 = 8$$

$$\text{Length scale factor} = \sqrt[3]{8} = 2$$

$$\text{Area scale factor} = 2^2 = 4$$

$$\text{Foil for the large egg} = 45 \times 4 = 180 \text{ cm}^2$$

Answer: 180 cm^2

Tip: Do not multiply 45 by 8. That is the volume scale factor, but foil covers the surface, so you need the area scale factor.

Marks: 1 mark for $216 \div 27$ or 8. 1 mark for the area scale factor 4. 1 mark for 180.

9 (3 marks)

A company makes two sizes of triangular warning sign.

The signs are mathematically similar.

The smaller sign is 60 cm tall and has an area of 1560 cm^2

The larger sign has an area of 3510 cm^2

Work out the height of the larger sign.

$$\text{Area scale factor} = 3510 \div 1560 = 2.25$$

Height is a length, so square root:

$$\text{Length scale factor} = \sqrt{2.25} = 1.5$$

Height of the larger sign = $60 \times 1.5 = 90$ cm

Answer: 90 cm

Tip: 2.25 is the area scale factor. Square root it before you use it on a height.

Marks: 1 mark for $3510 \div 1560$. 1 mark for the square root. 1 mark for 90.

10 (5 marks)

A small photo print is similar to a large photo print.

The large print is 50 cm wide and the small print is 20 cm wide.

The area of the large print is 3000 cm^2

Printing costs £25 per square metre.

Work out the cost of printing the small print.

Length scale factor = $20 \div 50 = 0.4$

Area scale factor = $0.4^2 = 0.16$

Area of small print = $3000 \times 0.16 = 480 \text{ cm}^2$

$1 \text{ m}^2 = 100 \times 100 = 10\,000 \text{ cm}^2$

So $480 \text{ cm}^2 = 480 \div 10\,000 = 0.048 \text{ m}^2$

Cost = $0.048 \times 25 = \text{£}1.20$

Answer: £1.20

Tip: There are $10\,000 \text{ cm}^2$ in 1 m^2 , not 100. Square the 100 because it is an area.

Marks: 1 mark for 0.4 or 2.5. 1 mark for squaring your scale factor. 1 mark for 480. 1 mark for dividing by 10 000. 1 mark for £1.20.

11 (4 marks)

Two cylindrical tins are mathematically similar.
The small tin has radius 4 cm and height 10 cm.
The large tin has volume $4320\pi \text{ cm}^3$
Work out the height of the large tin.

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$\text{Volume of the small tin} = \pi \times 4^2 \times 10 = 160\pi \text{ cm}^3$$

$$\text{Volume scale factor} = 4320\pi \div 160\pi = 27$$

$$\text{Length scale factor} = \sqrt[3]{27} = 3$$

$$\text{Height of the large tin} = 10 \times 3 = 30 \text{ cm}$$

Answer: 30 cm

Tip: 27 is the volume scale factor. Cube root it to get the length scale factor before you multiply the height. $10 \times 27 = 270$ is a common wrong answer.

Marks: 1 mark for 160π . 1 mark for the volume scale factor 27. 1 mark for $\sqrt[3]{27} = 3$. 1 mark for 30.
