

Recurring Decimals to Fractions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Convert each fraction to a recurring decimal.

(a) $\frac{5}{11}$

Divide 5 by 11 (short division):

$$5 \div 11 = 0.454545 \dots$$

The block 45 repeats, so put a dot over the 4 and the 5.

Answer: $0.\dot{4}\dot{5}$

(b) $\frac{7}{12}$

Divide 7 by 12 (short division):

$$7 \div 12 = 0.58333 \dots$$

Only the 3 repeats, so the dot goes over the 3 only.

Answer: $0.58\dot{3}$

Tip: Put dots only over the digits that repeat. $0.\dot{5}\dot{8}\dot{3}$ would mean $0.583583 \dots$, which is wrong.

Marks: 1 mark for 0.583 or a correct division method. 1 mark for $0.58\dot{3}$.

2 (1 mark)

Circle the fraction that is equal to $0.\dot{3}$

Options: $\frac{3}{10}$, $\frac{1}{3}$, $\frac{33}{100}$, $\frac{3}{100}$

$0.\dot{3} = 0.3333 \dots$ (the 3s go on forever)

$$\frac{1}{3} = 1 \div 3 = 0.3333 \dots \checkmark$$

$\frac{3}{10} = 0.3$, $\frac{33}{100} = 0.33$ and $\frac{3}{100} = 0.03$ all stop.

Answer: $\frac{1}{3}$

3 (1 mark)

Which of these fractions is equal to a recurring decimal?

Circle your answer.

Options: $\frac{3}{8}$, $\frac{7}{20}$, $\frac{2}{15}$, $\frac{9}{25}$

A fraction in its simplest form gives a terminating decimal only if the denominator has no prime factors other than 2 and 5.

$8 = 2^3$, $20 = 2^2 \times 5$, $25 = 5^2$, so these terminate.

$15 = 3 \times 5$ has a prime factor of 3, so $\frac{2}{15}$ recurs.

(Check: $\frac{2}{15} = 0.1333 \dots$)

Answer: $\frac{2}{15}$

4 (2 marks)

Prove that $0.\dot{27} = \frac{3}{11}$

Let $x = 0.272727 \dots$

The repeating block (27) has 2 digits, so multiply by 100:

$$100x = 27.272727 \dots$$

Subtract to remove the recurring part:

$$100x - x = 27.2727 \dots - 0.2727 \dots$$

$$99x = 27$$

$$x = \frac{27}{99}$$

Simplify (divide top and bottom by 9): $x = \frac{3}{11}$ ✓

Answer: $99x = 27$, so $x = \frac{27}{99} = \frac{3}{11}$

Tip: In a proof, start with “Let $x = 0.2727 \dots$ ” and show the recurring digits (with dots or “...”). Line up the decimal points when you subtract.

Marks: 1 mark for writing $100x = 27.2727 \dots$ and subtracting. 1 mark for $99x = 27$ and simplifying $\frac{27}{99}$ to $\frac{3}{11}$.

5 (3 marks)

Convert $0.4\dot{6}$ to a fraction in its simplest form.

Let $x = 0.4666 \dots$ (only the 6 repeats)

$$10x = 4.666 \dots$$

$$100x = 46.666 \dots$$

These two have the same digits after the decimal point, so subtract them:

$$100x - 10x = 46.666 \dots - 4.666 \dots$$

$$90x = 42$$

$$x = \frac{42}{90}$$

Simplify (divide top and bottom by 6): $x = \frac{7}{15}$

Answer: $\frac{7}{15}$

Tip: $0.4\dot{6}$ means $0.4666 \dots$, not $0.4646 \dots$. Write out the first few digits before you start.

Marks: 1 mark for two correct multiples, e.g. $10x = 4.666 \dots$ and $100x = 46.666 \dots$. 1 mark for subtracting to get $90x = 42$. 1 mark for $\frac{7}{15}$.

6 (3 marks)

Convert $0.2\dot{3}6$ to a fraction in its simplest form.

Let $x = 0.23636 \dots$ (the block 36 repeats)

$$10x = 2.3636 \dots$$

$$1000x = 236.3636 \dots$$

$$\text{Subtract: } 1000x - 10x = 236.3636 \dots - 2.3636 \dots$$

$$990x = 234$$

$$x = \frac{234}{990}$$

Simplify (divide top and bottom by 18): $x = \frac{13}{55}$

Answer: $\frac{13}{55}$

Marks: 1 mark for two correct multiples, e.g. $10x$ and $1000x$. 1 mark for $990x = 234$. 1 mark for $\frac{13}{55}$.

7 (1 mark)

Work out $0.2\dot{5} + 0.13$

Circle your answer.

Options: 0.3825 , $0.3\dot{8}$, $0.382\dot{5}$, $0.382\dot{5}$

Write both numbers to several decimal places and add in columns:

$$0.252525 \dots + 0.130000 \dots = 0.382525 \dots$$

After 0.38, the digits 2 and 5 keep repeating.

Answer: $0.38\dot{2}\dot{5}$

8 (4 marks)

You are given that $\frac{1}{11} = 0.\dot{0}\dot{9}$

(a) Write $0.\dot{6}\dot{3}$ as a fraction in its simplest form.

$$0.\dot{6}\dot{3} = 0.636363 \dots$$

This is 7 lots of $0.090909 \dots$, because $7 \times 0.0909 \dots = 0.6363 \dots$

$$\text{So } 0.\dot{6}\dot{3} = 7 \times \frac{1}{11} = \frac{7}{11}$$

Answer: $\frac{7}{11}$

(b) Hence, or otherwise, write $0.3\dot{4}\dot{5}$ as a fraction in its simplest form.

$$\text{Split the decimal: } 0.3\dot{4}\dot{5} = 0.3 + 0.0\dot{4}\dot{5}$$

$$0.\dot{4}\dot{5} = 0.4545 \dots = 5 \times 0.0909 \dots = \frac{5}{11}$$

$$0.0\dot{4}\dot{5} \text{ is this divided by 10: } \frac{5}{11} \div 10 = \frac{5}{110} = \frac{1}{22}$$

$$\text{Add: } \frac{3}{10} + \frac{1}{22} = \frac{33}{110} + \frac{5}{110} = \frac{38}{110}$$

$$\text{Simplify (divide top and bottom by 2): } \frac{19}{55}$$

(Or: $1000x - 10x = 345.4545 \dots - 3.4545 \dots$, so $990x = 342$ and $x = \frac{342}{990} = \frac{19}{55}$)

Answer: $\frac{19}{55}$

Tip: Only the 4 and the 5 repeat: $0.3\dot{4}\dot{5} = 0.34545 \dots$ Keep the 0.3 separate, or use $10x$ and $1000x$ so the recurring parts line up.

Marks: 1 mark for $\frac{3}{10} + \frac{5}{110}$ (or $990x = 342$). 1 mark for $\frac{38}{110}$ (or $\frac{342}{990}$). 1 mark for $\frac{19}{55}$.

9 (3 marks)

Work out $0.\dot{1} + 0.\dot{2}\dot{7}$

Give your answer as a fraction in its simplest form.

$$0.\dot{1} = \frac{1}{9} \text{ (just as } 0.\dot{8} = \frac{8}{9}\text{)}$$

For $0.\dot{2}\dot{7}$: let $y = 0.2727 \dots$, so $100y = 27.2727 \dots$

$$99y = 27, \text{ so } y = \frac{27}{99}$$

Use a common denominator of 99: $\frac{1}{9} = \frac{11}{99}$

$$\frac{11}{99} + \frac{27}{99} = \frac{38}{99}$$

38 and 99 have no common factor, so this is in its simplest form.

Answer: $\frac{38}{99}$

Tip: To add $\frac{1}{9}$ and $\frac{27}{99}$ you need a common denominator. Do not add the tops and the bottoms.

Marks: 1 mark for one of the decimals as a fraction. 1 mark for both. 1 mark for $\frac{38}{99}$.

10 (5 marks)

Work out $(0.\dot{3} + 0.1\dot{6}) \div 0.\dot{4}\dot{5}$

Give your answer as a fraction in its simplest form.

Change each recurring decimal to a fraction first.

$$0.\dot{3} = 0.333... = \frac{1}{3}$$

Let $x = 0.1666...$, so $10x = 1.666...$ and $100x = 16.666...$

$$100x - 10x = 15, \text{ so } 90x = 15 \text{ and } x = \frac{15}{90} = \frac{1}{6}$$

Let $y = 0.4545...$, so $100y = 45.4545...$

$$100y - y = 45, \text{ so } 99y = 45 \text{ and } y = \frac{45}{99} = \frac{5}{11}$$

$$\text{Add the first two: } \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Divide: } \frac{1}{2} \div \frac{5}{11} = \frac{1}{2} \times \frac{11}{5} = \frac{11}{10}$$

Check with decimals: $0.5 \div 0.4545... = 1.1$ ✓

Answer: $\frac{11}{10}$

Tip: In $0.1\dot{6}$ only the 6 repeats, so $10x = 1.666...$ (not $1.616...$). To divide by a fraction, multiply by its reciprocal.

Marks: 1 mark for a correct method for $0.1\dot{6}$ (e.g. $90x = 15$). 1 mark for $\frac{1}{6}$ and $\frac{1}{3}$. 1 mark for $\frac{5}{11}$. 1 mark for $\frac{1}{2} \div \frac{5}{11}$ or $\frac{1}{2} \times \frac{11}{5}$. 1 mark for $\frac{11}{10}$.