

Recognising and Sketching Graphs

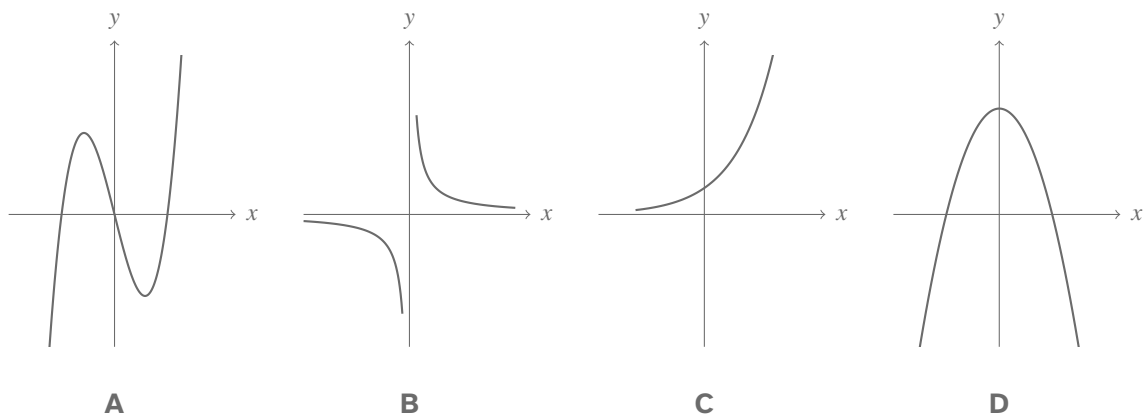
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Here are four graphs.



Write down the letter of the graph for each equation.

(a) $y = \frac{1}{x}$

This is a reciprocal graph.

It has two separate branches that never touch the axes.

Answer: B

(b) $y = 4 - x^2$

This is a quadratic with $-x^2$, so it is an upside-down U-shape.

It crosses the y-axis at 4.

Answer: D

(c) $y = x^3 - 4x$

This is a cubic, so it has an S-shape.

Answer: A

(d) $y = 2^x$

This is an exponential graph.

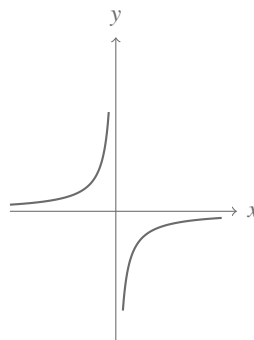
It passes through (0, 1) and is always above the x -axis.

Answer: C

Tip: Learn the basic shapes: straight line, quadratic (U or $\cap$), cubic (S-shape), reciprocal (two branches) and exponential.

2 (2 marks)

(a) Here is the sketch of a graph.



Circle the equation of the graph.

Options: $y = \frac{1}{x}$, $y = -x^3$, $y = -\frac{1}{x}$, $y = -x$

The graph has two separate branches that never touch the axes, so it is a reciprocal graph.

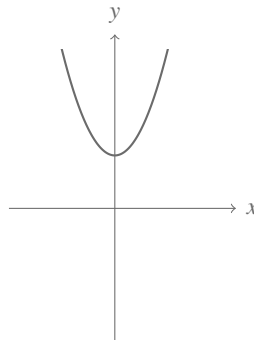
The branches are in the top-left and bottom-right.

So y is negative when x is positive: the equation is $y = -\frac{1}{x}$.

Answer: $y = -\frac{1}{x}$

Tip: $y = \frac{1}{x}$ has its branches in the top-right and bottom-left. The minus sign reflects the graph in the x -axis.

(b) Here is the sketch of a different graph.



Circle the possible equation of the graph.

Options: $y = x^2 - 2$, $y = (x + 2)^2$, $y = x^2 + 2$, $y = 2 - x^2$

The graph is a U-shape, so it is a quadratic with $+x^2$. That rules out $y = 2 - x^2$.

Its lowest point is on the y -axis, above the origin.

$y = x^2 - 2$ has its lowest point below the origin, and $y = (x + 2)^2$ touches the x -axis at $x = -2$.

So the equation is $y = x^2 + 2$.

Answer: $y = x^2 + 2$

3 (1 mark)

Which of these could be the equation of a graph that passes through (0,1) and never crosses the x -axis?

Circle your answer.

Options: $y = x^2 - 1$, $y = 3^x$, $y = \frac{1}{x}$, $y = x^3 + 1$

When $x = 0$: $3^0 = 1$, so $y = 3^x$ passes through (0,1).

3^x is always positive, so the graph never crosses the x -axis.

($y = x^2 - 1$ goes through (0, -1), $y = \frac{1}{x}$ has no y -intercept and $y = x^3 + 1$ crosses the x -axis at $x = -1$.)

Answer: $y = 3^x$

4 (2 marks)

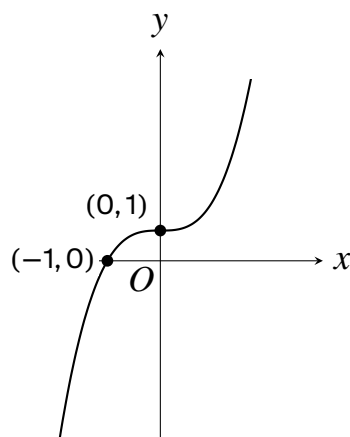
On the axes, sketch the curve $y = x^3 + 1$. Label the coordinates of the points where the curve crosses the x -axis and the y -axis.

$y = x^3 + 1$ is the graph of $y = x^3$ moved up by 1.

y -axis: when $x = 0$, $y = 0 + 1 = 1$, so it crosses at (0,1).

x -axis: when $y = 0$, $x^3 + 1 = 0$, so $x^3 = -1$ and $x = -1$. It crosses at (-1,0).

Draw a cubic S-shape, going up from left to right, that flattens out as it passes through (0,1).



Answer: Cubic shape through $(-1, 0)$ and $(0, 1)$

Tip: Draw one smooth S-shaped curve. A U-shape is wrong. Write each point with x first: the y -intercept is $(0, 1)$, not $(1, 0)$.

Marks: 1 mark for a cubic shape (going up, flattening at the y -axis). 1 mark for $(-1, 0)$ and $(0, 1)$ marked.

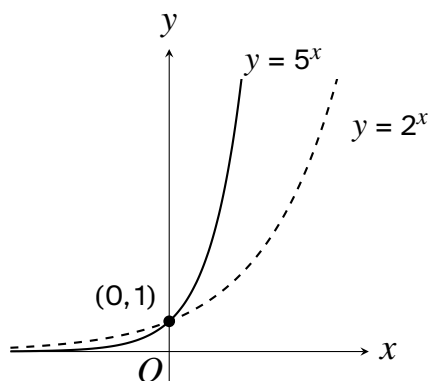
5 (2 marks)

Here is a sketch of the curve $y = 2^x$. On the same axes, sketch the curve $y = 5^x$

Both curves pass through $(0, 1)$, because $5^0 = 2^0 = 1$.

For $x > 0$, 5^x is bigger than 2^x , so the new curve is above (steeper).

For $x < 0$, 5^x is smaller than 2^x , so the new curve is below $y = 2^x$, but still above the x -axis.



Answer: A curve through $(0, 1)$; above $y = 2^x$ for $x > 0$, below it for $x < 0$

Tip: Many students drew $y = 5^x$ above $y = 2^x$ everywhere. Both curves cross at $(0, 1)$, and neither touches the x -axis.

Marks: 1 mark for a curve through $(0, 1)$. 1 mark for being steeper for $x > 0$ and closer to the x -axis for $x < 0$.

6 (1 mark)

Ffion sketches the graph of $y = \frac{1}{x}$ for $x > 0$. Her sketch starts on the y -axis. Make one criticism of her sketch.

Her curve starts on the y -axis, so it has a value when $x = 0$.

But $\frac{1}{0}$ cannot be worked out, so $y = \frac{1}{x}$ has no point with $x = 0$.

As x gets closer to 0, y gets bigger and bigger, so the curve should get closer to the y -axis without ever touching it.

Answer: The curve should not meet the y -axis, because $y = \frac{1}{x}$ has no value when $x = 0$.

Tip: Be precise. Say exactly what is wrong and why (e.g. “it touches the y -axis, but you cannot divide by 0”). Vague comments such as “the curve is wrong” score nothing.

7 (3 marks)

Sketch the graph of $y = (x - 1)(x + 3)$

Label the coordinates of the points where the graph meets the axes.

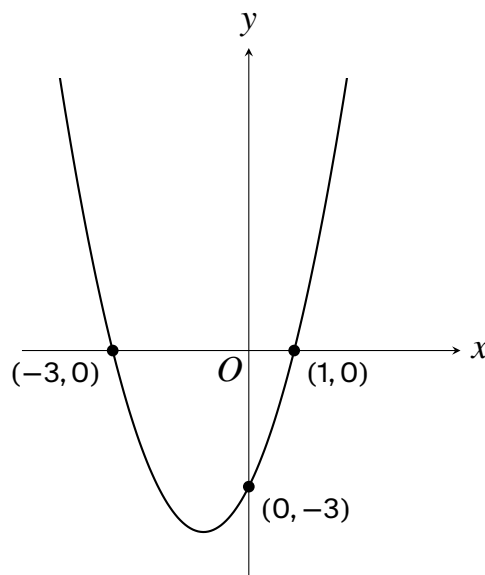
It is a quadratic with $+x^2$, so it is a U-shape.

x -intercepts: when $y = 0$, $x - 1 = 0$ or $x + 3 = 0$

So it meets the x -axis at $(1, 0)$ and $(-3, 0)$.

y -intercept: when $x = 0$, $y = (-1) \times 3 = -3$, so $(0, -3)$.

The lowest point is halfway between the roots, at $x = -1$: $y = (-2)(2) = -4$



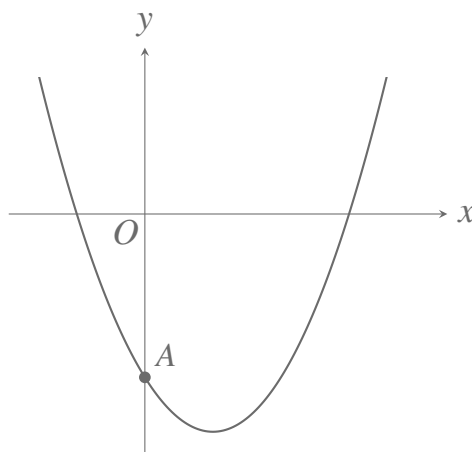
Answer: U-shape through $(1, 0)$, $(-3, 0)$ and $(0, -3)$

Tip: The root from $(x + 3)$ is $x = -3$, not $x = 3$. Set each bracket equal to 0.

Marks: 1 mark for a U-shape. 1 mark for both x -intercepts. 1 mark for the y -intercept.

8 (3 marks)

Here is a sketch of the curve $y = (x + 2)(x - 6)$



- (a) The curve meets the y -axis at A .
Work out the coordinates of A .

On the y -axis, $x = 0$.

$$y = (0 + 2)(0 - 6) = 2 \times (-6) = -12$$

Answer: $(0, -12)$

- (b) The line $y = k$ meets the curve at exactly one point.
Work out the value of k .

$y = k$ is a horizontal line. It meets the U-shaped curve at exactly one point only at the turning point.

The curve crosses the x -axis at $x = -2$ and $x = 6$.

The turning point is halfway between them: $x = \frac{-2 + 6}{2} = 2$

Substitute $x = 2$: $y = (2 + 2)(2 - 6) = 4 \times (-4) = -16$

So $k = -16$.

Answer: $k = -16$

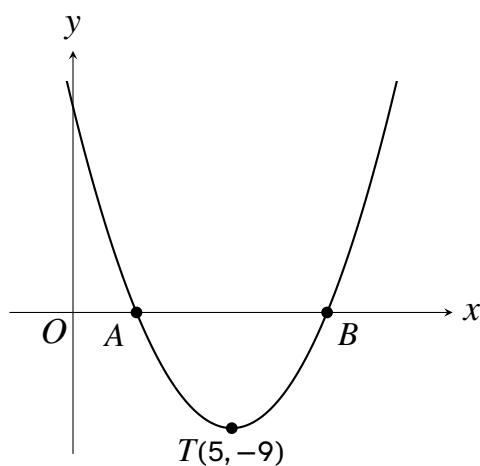
Tip: The turning point is halfway between the roots. Here the roots are -2 and 6 , not 2 and -6 .

Marks: 1 mark for $x = 2$ (or a method for the turning point). 1 mark for -16 .

9 (4 marks)

Here is a sketch of $y = x^2 + px + q$

$T(5, -9)$ is the turning point of the curve.



Not drawn accurately

- (a) Work out the value of p and the value of q .

The turning point is $(5, -9)$, so in completed-square form the curve is $y = (x - 5)^2 - 9$.

Expand: $(x - 5)^2 - 9 = x^2 - 10x + 25 - 9$

$= x^2 - 10x + 16$

Compare with $x^2 + px + q$.

Answer: $p = -10$, $q = 16$

Tip: A turning point at $(5, -9)$ gives $(x - 5)^2 - 9$: the sign inside the bracket is the opposite of the x -coordinate.

Marks: 1 mark for $(x - 5)^2 - 9$. 1 mark for both values.

- (b) The curve crosses the x -axis at A and B .
Work out the length AB .

At A and B , $y = 0$: $x^2 - 10x + 16 = 0$

Factorise: $(x - 2)(x - 8) = 0$

So $x = 2$ or $x = 8$.

$AB = 8 - 2 = 6$

(Or: $(x - 5)^2 = 9$, so $x - 5 = \pm 3$. The points are 3 either side of $x = 5$.)

Answer: 6

Marks: 1 mark for $x = 2$ and $x = 8$ (or 5 ± 3). 1 mark for 6.
