

Rationalising the Denominator

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (6 marks)

Rationalise the denominator and simplify fully

(a) $\frac{21}{\sqrt{3}}$

Multiply the top and the bottom by $\sqrt{3}$:

$$\frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} \quad (\text{because } \sqrt{3} \times \sqrt{3} = 3)$$

Simplify: $21 \div 3 = 7$, so the answer is $7\sqrt{3}$

Answer: $7\sqrt{3}$

Tip: "Simplify fully" means you must cancel at the end. Do not leave the answer as $\frac{21\sqrt{3}}{3}$.

Marks: 1 mark for $\frac{21\sqrt{3}}{3}$. 1 mark for $7\sqrt{3}$.

(b) $\frac{22}{\sqrt{11}}$

Multiply the top and the bottom by $\sqrt{11}$:

$$\frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}} = \frac{22\sqrt{11}}{11}$$

Simplify: $22 \div 11 = 2$, so the answer is $2\sqrt{11}$

Answer: $2\sqrt{11}$

Marks: 1 mark for $\frac{22\sqrt{11}}{11}$. 1 mark for $2\sqrt{11}$.

(c) $\frac{1}{2\sqrt{7}}$

Only the $\sqrt{7}$ needs removing, so multiply the top and the bottom by $\sqrt{7}$:

$$\begin{aligned}\frac{1}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} &= \frac{\sqrt{7}}{2 \times 7} \\ &= \frac{\sqrt{7}}{14}\end{aligned}$$

Answer: $\frac{\sqrt{7}}{14}$

Marks: 1 mark for multiplying the top and bottom by $\sqrt{7}$. 1 mark for $\frac{\sqrt{7}}{14}$.

2 (2 marks)

Show that $\frac{28}{\sqrt{14}} = \sqrt{56}$

Multiply the top and the bottom by $\sqrt{14}$:

$$\frac{28}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{28\sqrt{14}}{14} = 2\sqrt{14}$$

Write 2 as $\sqrt{4}$: $2\sqrt{14} = \sqrt{4} \times \sqrt{14} = \sqrt{56}$ ✓

Answer: $\frac{28\sqrt{14}}{14} = 2\sqrt{14} = \sqrt{56}$

Tip: To put a number inside a square root, square it first: $2\sqrt{14} = \sqrt{2^2 \times 14}$, not $\sqrt{28}$.

Marks: 1 mark for $\frac{28\sqrt{14}}{14}$ or $2\sqrt{14}$. 1 mark for showing $2\sqrt{14} = \sqrt{4 \times 14} = \sqrt{56}$.

3 (1 mark)

Circle the expression equal to $\frac{12}{\sqrt{8}}$

Options: $3\sqrt{2}$, $\frac{3}{2}$, $4\sqrt{2}$, $\frac{3\sqrt{2}}{2}$

Simplify the surd first: $\sqrt{8} = 2\sqrt{2}$

$$\frac{12}{2\sqrt{2}} = \frac{6}{\sqrt{2}}$$

Multiply the top and the bottom by $\sqrt{2}$: $\frac{6\sqrt{2}}{2} = 3\sqrt{2}$

Answer: $3\sqrt{2}$

4 (2 marks)

Write $\frac{\sqrt{3}+1}{\sqrt{3}}$ in the form $\frac{a+\sqrt{b}}{c}$ where a , b and c are integers.

Multiply the top and the bottom by $\sqrt{3}$:

Top: $(\sqrt{3}+1) \times \sqrt{3} = 3 + \sqrt{3}$

Bottom: $\sqrt{3} \times \sqrt{3} = 3$

So the answer is $\frac{3+\sqrt{3}}{3}$

Answer: $\frac{3+\sqrt{3}}{3}$

Tip: Multiply **both** terms on the top by $\sqrt{3}$: $\sqrt{3} \times \sqrt{3} = 3$ and $1 \times \sqrt{3} = \sqrt{3}$.

Marks: 1 mark for multiplying the top and bottom by $\sqrt{3}$. 1 mark for $\frac{3+\sqrt{3}}{3}$.

5 (3 marks)

A triangle has area 10 cm^2 and base $\sqrt{5} \text{ cm}$.

Work out the height of the triangle.

Give your answer in the form $k\sqrt{5}$

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$\frac{1}{2} \times \sqrt{5} \times h = 10$$

Multiply both sides by 2: $\sqrt{5} \times h = 20$

$$h = \frac{20}{\sqrt{5}}$$

Rationalise: $\frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$

Answer: $4\sqrt{5}$ cm

Marks: 1 mark for using the area formula to get $h = \frac{20}{\sqrt{5}}$. 1 mark for rationalising. 1 mark for $4\sqrt{5}$.

6 (3 marks)

Simplify fully $\frac{\sqrt{20} + \sqrt{45}}{\sqrt{10}}$

Give your answer in the form $\frac{a\sqrt{2}}{b}$

Simplify the top first:

$$\sqrt{20} = 2\sqrt{5} \text{ and } \sqrt{45} = 3\sqrt{5}$$

$$\text{So the top is } 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$$

$$\text{Now } \frac{5\sqrt{5}}{\sqrt{10}} = \frac{5\sqrt{5}}{\sqrt{5} \times \sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$\text{Rationalise: } \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

Answer: $\frac{5\sqrt{2}}{2}$

Tip: Simplify the surds on the top before anything else. $\sqrt{20} + \sqrt{45}$ is not $\sqrt{65}$.

Marks: 1 mark for $5\sqrt{5}$. 1 mark for a correct method to rationalise. 1 mark for $\frac{5\sqrt{2}}{2}$.

7 (3 marks)

Show that $\frac{3\sqrt{2}}{\sqrt{5}} - \frac{\sqrt{10}}{10}$ can be written in the form $\frac{\sqrt{a}}{b}$ where a and b are integers.

Rationalise the first fraction:

$$\frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$$

Make the denominator 10 to match the second fraction:

$$\frac{3\sqrt{10}}{5} = \frac{6\sqrt{10}}{10}$$

$$\text{Subtract: } \frac{6\sqrt{10}}{10} - \frac{\sqrt{10}}{10} = \frac{5\sqrt{10}}{10}$$

$$\text{Simplify: } \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2} \quad \checkmark$$

Answer: $\frac{\sqrt{10}}{2}$

Tip: Use a common denominator of 10, just like with ordinary fractions. Do not add the denominators (5 and 10) to get 15.

Marks: 1 mark for $\frac{3\sqrt{10}}{5}$. 1 mark for a common denominator, e.g. $\frac{6\sqrt{10}}{10} - \frac{\sqrt{10}}{10}$. 1 mark for $\frac{\sqrt{10}}{2}$.

8 (4 marks)

Work out $\sqrt{20} - \frac{36}{\sqrt{80}}$

Give your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are integers.

Simplify the surds: $\sqrt{20} = 2\sqrt{5}$ and $\sqrt{80} = 4\sqrt{5}$

$$\frac{36}{4\sqrt{5}} = \frac{9}{\sqrt{5}}$$

$$\text{Rationalise: } \frac{9}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{9\sqrt{5}}{5}$$

$$\text{Write } 2\sqrt{5} \text{ over } 5: 2\sqrt{5} = \frac{10\sqrt{5}}{5}$$

$$\text{Subtract: } \frac{10\sqrt{5}}{5} - \frac{9\sqrt{5}}{5} = \frac{\sqrt{5}}{5}$$

Answer: $\frac{\sqrt{5}}{5}$

Marks: 1 mark for $2\sqrt{5}$ or $4\sqrt{5}$. 1 mark for rationalising, e.g. $\frac{9\sqrt{5}}{5}$ or $\frac{36\sqrt{80}}{80}$. 1 mark for using a common denominator. 1 mark for $\frac{\sqrt{5}}{5}$.
