

Ratio and Algebra

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

$$y:x = 3:2$$

Circle the equation linking x and y .

Options: $y = \frac{2}{3}x$, $y = \frac{3}{2}x$, $2y = 3x + 1$, $3y = 2x$

$$y:x = 3:2 \text{ means } \frac{y}{x} = \frac{3}{2}$$

Multiply both sides by x : $y = \frac{3}{2}x$

Check with $x = 2$: $y = \frac{3}{2} \times 2 = 3$ ✓

Answer: $y = \frac{3}{2}x$

Tip: Test your choice with the numbers in the ratio: $y = 3$ and $x = 2$ must fit the equation.

2 (1 mark)

$$\frac{x}{4} = \frac{y}{6}$$

Circle the ratio $y:x$

Options: $2:3$, $3:2$, $24:1$, $1:24$

Multiply both sides by 12: $3x = 2y$

Try $x = 2$: then $2y = 6$, so $y = 3$.

So $y:x = 3:2$

Check: $\frac{2}{4} = \frac{1}{2}$ and $\frac{3}{6} = \frac{1}{2}$ ✓

Answer: 3 : 2

Tip: Read the order carefully: the question asks for $y:x$, not $x:y$.

3 (2 marks)

$$p:q = 3:8 \text{ and } 2p + q = 42$$

Work out $q - p$.

$$\text{Let } p = 3k \text{ and } q = 8k.$$

$$2p + q = 6k + 8k = 14k$$

$$14k = 42, \text{ so } k = 3$$

$$p = 9 \text{ and } q = 24$$

$$q - p = 24 - 9 = 15$$

Answer: 15

Tip: $2p$ is worth $2 \times 3 = 6$ parts, not 3 parts. So $2p + q$ is $6 + 8 = 14$ parts.

Marks: 1 mark for $14k = 42$ (or $p = 9$ and $q = 24$). 1 mark for 15.

4 (2 marks)

$$x:y = 2:5$$

Show that $(x + y):(y - x) = 7:3$

$$x:y = 2:5, \text{ so let } x = 2k \text{ and } y = 5k.$$

$$x + y = 2k + 5k = 7k$$

$$y - x = 5k - 2k = 3k$$

$$\text{So } (x + y):(y - x) = 7k:3k$$

Divide both parts by k : 7:3

Answer: Shown

Marks: 1 mark for $x = 2k$ and $y = 5k$ (or using a correct pair of values, such as 2 and 5). 1 mark for a complete method that reaches 7:3.

5 (3 marks)

$$(x + 3) : (2x - 1) = 4 : 5$$

Work out the value of x .

Write the ratios as fractions: $\frac{x + 3}{2x - 1} = \frac{4}{5}$

$$\text{Cross-multiply: } 5(x + 3) = 4(2x - 1)$$

$$5x + 15 = 8x - 4$$

$$19 = 3x$$

$$x = \frac{19}{3}$$

Answer: $\frac{19}{3}$

Tip: When you cross-multiply, each bracket goes with the number on the **other** side.

$4(x + 3) = 5(2x - 1)$ is a common mistake.

Marks: 1 mark for $5(x + 3) = 4(2x - 1)$. 1 mark for $5x + 15 = 8x - 4$ or $3x = 19$. 1 mark for $\frac{19}{3}$.

6 (3 marks)

$$3a = 2b$$

$$b + c = 2a$$

Work out the ratio $a : b : c$

Give your answer in its simplest form.

$$3a = 2b, \text{ so } b = \frac{3}{2}a$$

$$b + c = 2a, \text{ so } c = 2a - b = 2a - \frac{3}{2}a = \frac{1}{2}a$$

Choose a value of a that makes b and c whole numbers. $a = 2$ works.

$$b = \frac{3}{2} \times 2 = 3$$

$$c = \frac{1}{2} \times 2 = 1$$

So $a:b:c = 2:3:1$

Check: $3 \times 2 = 2 \times 3$ ✓ and $3 + 1 = 2 \times 2$ ✓

Answer: 2 : 3 : 1

Tip: $3a = 2b$ gives $b = \frac{3a}{2}$, so $a:b = 2:3$, not $3:2$. Check your final ratio in both equations.

Marks: 1 mark for b in terms of a (or a in terms of b). 1 mark for c in terms of a (or a correct ratio in any form, e.g. 1:1.5:0.5). 1 mark for 2 : 3 : 1.

7 (3 marks)

$$\frac{2m + n}{m + 3n} = \frac{5}{4}$$

Work out the ratio $m:n$

Cross-multiply: $4(2m + n) = 5(m + 3n)$

$$8m + 4n = 5m + 15n$$

$$3m = 11n$$

$$\frac{m}{n} = \frac{11}{3}$$

So $m:n = 11:3$

Answer: 11 : 3

Tip: From $3m = 11n$ the numbers swap sides: $m:n = 11:3$. Writing 3:11 is a very common mistake. Check: $m = 11$, $n = 3$ gives $\frac{25}{20} = \frac{5}{4}$ ✓

Marks: 1 mark for $4(2m + n) = 5(m + 3n)$. 1 mark for $3m = 11n$. 1 mark for 11 : 3.

8 (3 marks)

In a bag, the ratio of red to blue counters is 2:3

10 red counters are added.

The ratio is now 4:3

How many blue counters are in the bag?

Let the number of red counters be $2x$ and blue be $3x$.

10 red are added: red = $2x + 10$. Blue stays $3x$.

$$\frac{2x + 10}{3x} = \frac{4}{3}$$

Cross-multiply: $3(2x + 10) = 4 \times 3x$

$$6x + 30 = 12x$$

$$30 = 6x$$

$$x = 5$$

$$\text{Blue} = 3 \times 5 = 15$$

Answer: 15

Marks: 1 mark for a correct equation. 1 mark for $x = 5$. 1 mark for 15.

9 (4 marks)

The ages of Tom and his dad are in the ratio 1:4

In 9 years' time the ratio will be 2:5

How old is Tom now?

Let Tom's age be x , so his dad's age is $4x$.

In 9 years: Tom is $x + 9$ and his dad is $4x + 9$.

$$\frac{x + 9}{4x + 9} = \frac{2}{5}$$

Cross-multiply: $5(x + 9) = 2(4x + 9)$

$$5x + 45 = 8x + 18$$

$$27 = 3x$$

$$x = 9$$

Check: now 9 and 36 (ratio 1:4), in 9 years 18 and 45 (ratio 2:5) ✓

Answer: 9 years

Marks: 1 mark for $x + 9$ and $4x + 9$. 1 mark for a correct equation. 1 mark for $3x = 27$. 1 mark for 9.

10 (4 marks)

Triangle T has base 32 cm and perpendicular height h cm.

Parallelogram R has base 18 cm and perpendicular height k cm.

The area of R is 20% more than the area of T .

Work out the ratio $h:k$

Give your answer in its simplest form.

$$\text{Area of } T = \frac{1}{2} \times 32 \times h = 16h$$

$$\text{Area of } R \text{ is 20\% more: } 1.2 \times 16h = 19.2h$$

$$\text{Area of } R \text{ is also base} \times \text{height} = 18k$$

$$\text{So } 18k = 19.2h$$

$$\frac{h}{k} = \frac{18}{19.2} = \frac{180}{192} = \frac{15}{16}$$

$$h:k = 15:16$$

Answer: 15 : 16

Tip: Remember the $\frac{1}{2}$ in the area of a triangle. Then check your ratio is the right way round:

$$h = 15, k = 16 \text{ gives } 18 \times 16 = 288 \text{ and } 1.2 \times 16 \times 15 = 288 \quad \checkmark$$

Marks: 1 mark for $16h$. 1 mark for $1.2 \times 16h$ (or $19.2h$). 1 mark for $18k = 19.2h$. 1 mark for 15 : 16.
