

Quadratic Inequalities

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Write down the solution of

(a) $x^2 < 25$

Solve $x^2 = 25$ first: $x = 5$ or $x = -5$

$x^2 < 25$ when x is between -5 and 5 .

Check: $x = 0$ gives $0 < 25$ ✓

Answer: $-5 < x < 5$

Tip: Don't forget the negative root. $x < 5$ is wrong: for example, $x = -6$ gives 36 , which is not less than 25 .

(b) $x^2 \geq 49$

Solve $x^2 = 49$ first: $x = 7$ or $x = -7$

$x^2 \geq 49$ when x is 7 or more, or when x is -7 or less.

Check: $x = -10$ gives $100 \geq 49$ ✓

Answer: $x \leq -7$ or $x \geq 7$

2 (3 marks)

Solve $x^2 - 5x + 6 \leq 0$

Factorise: $x^2 - 5x + 6 = (x - 2)(x - 3)$

The critical values (where it equals 0) are $x = 2$ and $x = 3$.

Sketch $y = x^2 - 5x + 6$: a U-shaped curve crossing the x -axis at 2 and 3.

≤ 0 means on or below the x -axis.

This happens between the two critical values.

Answer: $2 \leq x \leq 3$

Marks: 1 mark for factorising. 1 mark for the critical values 2 and 3. 1 mark for $2 \leq x \leq 3$.

3 (3 marks)

Solve $x^2 + 3x - 10 > 0$

Factorise: $x^2 + 3x - 10 = (x + 5)(x - 2)$

The critical values are $x = -5$ and $x = 2$.

Sketch $y = x^2 + 3x - 10$: a U-shaped curve crossing the x -axis at -5 and 2.

> 0 means above the x -axis.

This happens to the left of -5 and to the right of 2.

Answer: $x < -5$ or $x > 2$

Tip: Here the answer is two separate inequalities. Don't combine them into something like $-5 > x > 2$, which makes no sense.

Marks: 1 mark for factorising. 1 mark for the critical values -5 and 2. 1 mark for $x < -5$ or $x > 2$.

4 (2 marks)

How many integers satisfy $x^2 \leq 10$?

Solve $x^2 = 10$: $x = \pm\sqrt{10} = \pm 3.16 \dots$

So $-3.16 \dots \leq x \leq 3.16 \dots$

The integers in this range are $-3, -2, -1, 0, 1, 2, 3$.

That is 7 integers.

Answer: 7

Marks: 1 mark for $\pm\sqrt{10}$, or for -3 and 3 . 1 mark for 7 .

5 (4 marks)

Solve $2x^2 - x - 3 \geq 0$

Give your answer in set notation.

Factorise: $2x^2 - x - 3 = (2x - 3)(x + 1)$

Critical values:

$2x - 3 = 0$ gives $x = 1.5$

$x + 1 = 0$ gives $x = -1$

Sketch: a U-shaped curve crossing the x -axis at -1 and 1.5 .

≥ 0 means on or above the x -axis, which is outside the two roots.

So $x \leq -1$ or $x \geq 1.5$

In set notation, “or” is written with $\cup$ (union).

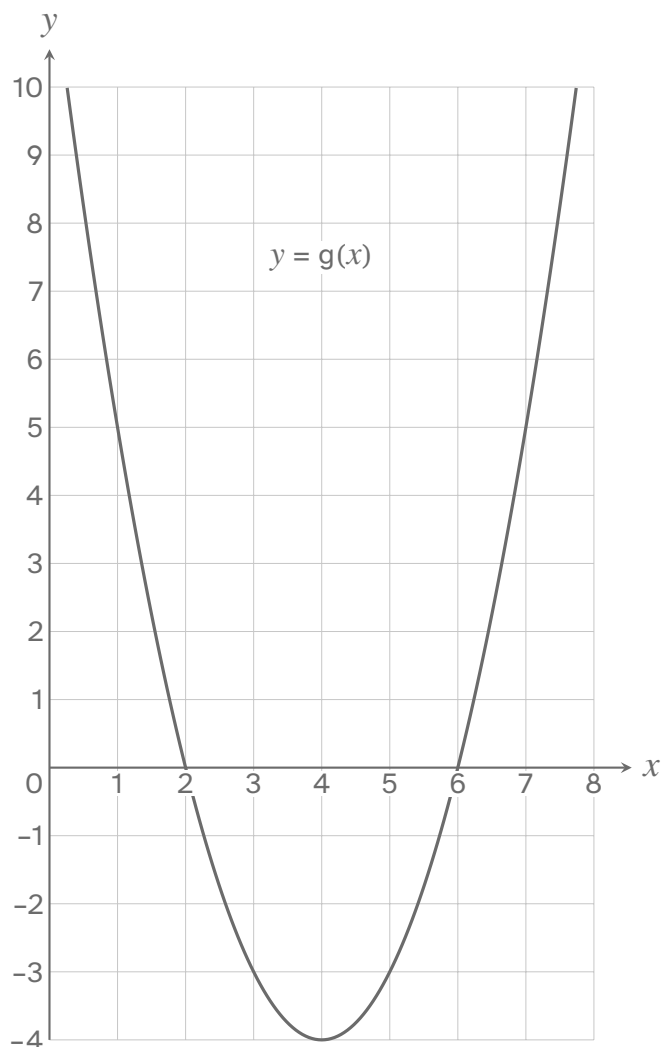
Answer: $\{x: x \leq -1\} \cup \{x: x \geq 1.5\}$

Tip: Use $\cup$ (union) to join the two parts. $\cap$ would mean both at the same time, which is impossible here.

Marks: 1 mark for factorising. 1 mark for the critical values -1 and 1.5 . 1 mark for $x \leq -1$ or $x \geq 1.5$. 1 mark for correct set notation.

6 (3 marks)

A curve $y = g(x)$ is drawn on the grid below.
 $g(x)$ is a quadratic expression.



- (a) Use the graph to work out how many integer values of x satisfy $g(x) \leq 0$

$g(x) \leq 0$ means the curve is on or below the x -axis.

The curve meets the x -axis at $x = 2$ and $x = 6$.

So $2 \leq x \leq 6$

The integers are 2, 3, 4, 5, 6 (2 and 6 are included, because $g(x) = 0$ there).

That is 5 integers.

Answer: 5

Tip: $\leq$ includes 0, so the points where the curve meets the x -axis count. Don't leave out 2 and 6.

Marks: 1 mark for $2 \leq x \leq 6$ (or listing the integers 2, 3, 4, 5, 6). 1 mark for 5.

(b) Circle the solution of $g(x) > 5$

Options: $1 < x < 7$, $x < 1$ or $x > 7$, $x > 7$, $x < 2$ or $x > 6$

Draw the line $y = 5$ across the graph.

The curve meets $y = 5$ at $x = 1$ and $x = 7$.

$g(x) > 5$ means the curve is above this line.

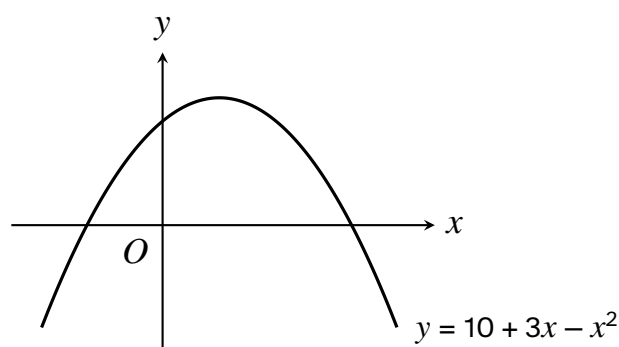
This is to the left of 1 and to the right of 7.

Answer: $x < 1$ or $x > 7$

Tip: $g(x) > 5$ is not about where the curve crosses the x -axis. Use the line $y = 5$ instead.

7 (3 marks)

Here is a sketch of the curve $y = 10 + 3x - x^2$



Not drawn accurately

Solve $10 + 3x - x^2 \geq 0$

Multiply by -1 and reverse the sign: $x^2 - 3x - 10 \leq 0$

Factorise: $x^2 - 3x - 10 = (x - 5)(x + 2)$

So the curve crosses the x -axis at $x = -2$ and $x = 5$.

The curve $y = 10 + 3x - x^2$ is $\cap$ -shaped (the x^2 term is negative).

≥ 0 means on or above the x -axis.

The $\cap$ -shaped curve is on or above the x -axis between -2 and 5 .

Answer: $-2 \leq x \leq 5$

Tip: This curve is $\cap$ -shaped, not U-shaped, so the part above the x -axis is **between** the roots. Use the sketch to check. If you multiply by -1 , remember to reverse the inequality sign.

Marks: 1 mark for factorising (e.g. $(5 - x)(2 + x)$ or $(x - 5)(x + 2)$). 1 mark for -2 and 5 . 1 mark for $-2 \leq x \leq 5$.

8 (3 marks)

The number line shows the solution of a quadratic inequality.



- (a) Write the solution shown on the number line as an inequality.

The circles are filled in, so -4 and 3 are included.

The solution is every value from -4 to 3 .

Answer: $-4 \leq x \leq 3$

Tip: Filled circles mean $\leq$ or $\geq$. Open circles mean $<$ or $>$.

- (b) Kasia says the number line shows the solution of $x^2 - x - 12 \leq 0$

Is Kasia correct? Tick a box.

You must show your working.

Factorise: $x^2 - x - 12 = (x - 4)(x + 3)$

The critical values are $x = 4$ and $x = -3$.

The U-shaped curve is on or below the x -axis between them, so the solution is $-3 \leq x \leq 4$.

This is not $-4 \leq x \leq 3$, so Kasia is wrong.

(The number line shows the solution of $x^2 + x - 12 \leq 0$, which factorises to $(x + 4)(x - 3) \leq 0$.)

Answer: No: $x^2 - x - 12 \leq 0$ gives $-3 \leq x \leq 4$

Tip: When you factorise, check the signs by expanding again: $(x - 4)(x + 3) = x^2 - x - 12$. It is easy to get -4 and 3 the wrong way round.

Marks: 1 mark for factorising to $(x - 4)(x + 3)$ or for the critical values 4 and -3 . 1 mark for No with $-3 \leq x \leq 4$.

9 (1 mark)

Priya has solved the inequality $x^2 > 9$ and represented her solution on the number line below.



Give **one** reason why her solution is wrong.

$x^2 > 9$ when $x > 3$, but also when $x < -3$.

For example, $x = -4$ gives $(-4)^2 = 16$, which is more than 9.

Priya has only shown $x > 3$.

Answer: She has missed out $x < -3$ (e.g. $x = -4$ gives $16 > 9$)

10 (4 marks)

Solve $3x^2 > 4 - 11x$

Get 0 on one side. Add $11x$ and subtract 4:

$$3x^2 + 11x - 4 > 0$$

Factorise: $(3x - 1)(x + 4) > 0$

Critical values:

$$3x - 1 = 0 \text{ gives } x = \frac{1}{3}$$

$$x + 4 = 0 \text{ gives } x = -4$$

The U-shaped curve $y = 3x^2 + 11x - 4$ is above the x -axis outside these values.

$$\textbf{Answer: } x < -4 \text{ or } x > \frac{1}{3}$$

Tip: Rearrange so that one side is 0 before you do anything else. Take care with signs when you move terms across.

Marks: 1 mark for $3x^2 + 11x - 4$. 1 mark for factorising (or using the formula). 1 mark for $\frac{1}{3}$ and -4 . 1 mark for both inequalities.
