

Proving Circle Theorems

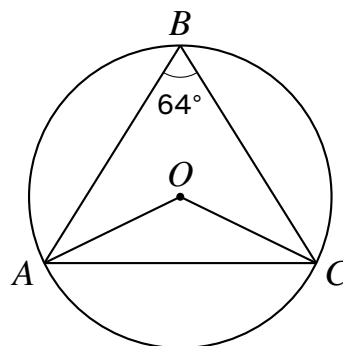
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

A, B and C are points on a circle, centre O.



Not drawn accurately

Angle $ABC = 64^\circ$

Jo works out the size of angle OAC . Complete the reasons in her working.

$OA = OC$ because they are both radii of the circle.

Angle $AOC = 128^\circ$ because the angle at the centre is twice the angle at the circumference ($2 \times 64 = 128$).

Angle $OAC = 26^\circ$ because triangle OAC is isosceles, so its base angles are equal.

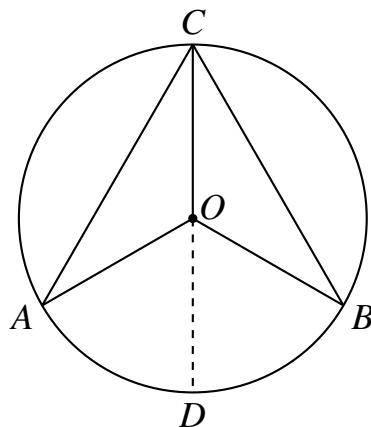
(Also, angles in a triangle add up to 180° .)

Tip: Write each reason in full. “Radius” or “isosceles” on its own is usually not enough; say what is equal and why.

Marks: 1 mark for “radii”. 1 mark for “the angle at the centre is twice the angle at the circumference”. 1 mark for “base angles of an isosceles triangle are equal”.

2 (4 marks)

A , B and C are points on a circle, centre O .



Not drawn accurately

Prove that the angle at the centre is twice the angle at the circumference, i.e. angle $AOB = 2 \times \text{angle } ACB$

You may use the diagram. CO is extended to D .

Let angle $OCA = x$ and angle $OCB = y$.

So angle $ACB = x + y$.

$OA = OC$ (radii), so triangle OAC is isosceles.

So angle $OAC = x$ (base angles of an isosceles triangle are equal).

Angle $AOD = x + x = 2x$

(The exterior angle of a triangle equals the sum of the two opposite interior angles.)

In the same way, $OB = OC$, so angle $OBC = y$ and angle $BOD = 2y$.

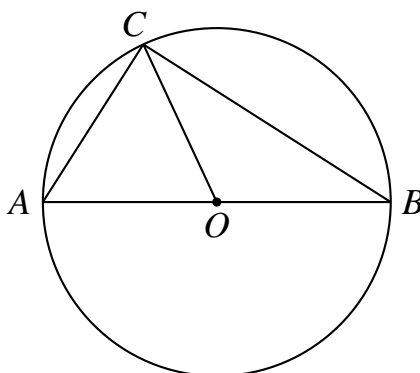
Angle $AOB = 2x + 2y = 2(x + y)$

So angle $AOB = 2 \times \text{angle } ACB$.

Marks: 1 mark for using the isosceles triangles. 1 mark for using the exterior angle (or angles in a triangle and on a straight line). 1 mark for $2x + 2y$. 1 mark for a complete proof.

3 (4 marks)

AB is a diameter of a circle, centre O . C is a point on the circle.



Not drawn accurately

Prove that angle $ACB = 90^\circ$

$OA = OC = OB$ (radii of the circle)

Triangle OAC is isosceles, so angle $OAC = \text{angle } OCA$. Call this a .

Triangle OBC is isosceles, so angle $OBC = \text{angle } OCB$. Call this b .

The angles of triangle ABC are a , b and $a + b$.

Angles in a triangle add up to 180° : $a + b + (a + b) = 180$

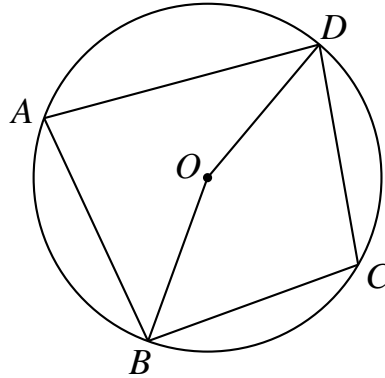
$2a + 2b = 180$, so $a + b = 90$

Angle $ACB = a + b = 90^\circ$

Marks: 1 mark for the radii being equal. 1 mark for using the isosceles triangles. 1 mark for using the angle sum of a triangle. 1 mark for a complete proof.

4 (3 marks)

$ABCD$ is a cyclic quadrilateral, centre O .



Not drawn accurately

Prove that opposite angles of a cyclic quadrilateral add up to 180°

You may use the fact that the angle at the centre is twice the angle at the circumference.

Let angle $BAD = x$.

The angle at the centre on the same arc is angle $BOD = 2x$.

(The angle at the centre is twice the angle at the circumference.)

Angles around a point add up to 360° , so reflex angle $BOD = 360 - 2x$.

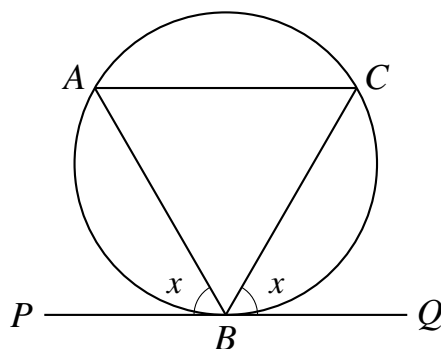
Angle BCD stands on the other arc, so angle $BCD = \frac{1}{2}(360 - 2x) = 180 - x$

Angle BAD + angle $BCD = x + (180 - x) = 180^\circ$

Marks: 1 mark for angle $BOD = 2x$. 1 mark for reflex angle $BOD = 360 - 2x$. 1 mark for completing the proof.

5 (3 marks)

A , B and C are points on a circle.



Not drawn accurately

PBQ is a tangent to the circle at B .

Angle ABP = angle CBQ = x

Prove that $AB = BC$

Angle ACB = angle ABP = x

(Alternate segment theorem.)

Angle BAC = angle CBQ = x

(Alternate segment theorem.)

So angle ACB = angle BAC .

So triangle ABC is isosceles.

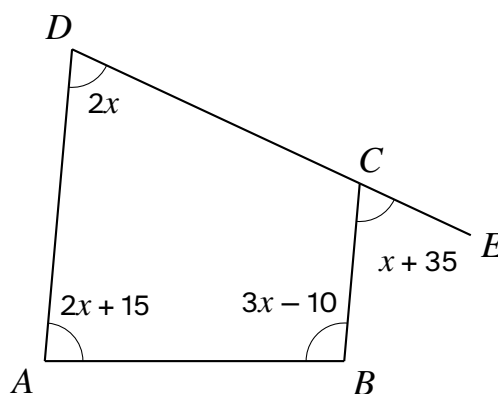
The sides opposite the equal angles are equal, so $AB = BC$.

Tip: Do not assume AC is parallel to PQ . Use the alternate segment theorem for each angle.

Marks: 1 mark for one angle in triangle ABC equal to x with a reason. 1 mark for the second angle with a reason. 1 mark for isosceles, so $AB = BC$.

6 (4 marks)

Here is a quadrilateral $ABCD$.



Not drawn accurately

DCE is a straight line.

The angles are in degrees.

Prove that $ABCD$ is **not** a cyclic quadrilateral.

Angles on a straight line add up to 180° , so angle $BCD = 180 - (x + 35) = 145 - x$

Angles in a quadrilateral add up to 360° :

$$(2x + 15) + (3x - 10) + (145 - x) + 2x = 360$$

$$6x + 150 = 360$$

$$6x = 210, \text{ so } x = 35$$

Angle $DAB = 2 \times 35 + 15 = 85^\circ$ and angle $BCD = 145 - 35 = 110^\circ$

(Also angle $ABC = 95^\circ$ and angle $CDA = 70^\circ$.)

A and C are opposite angles: $85 + 110 = 195$

$195 \neq 180$, so the opposite angles do not add up to 180° .

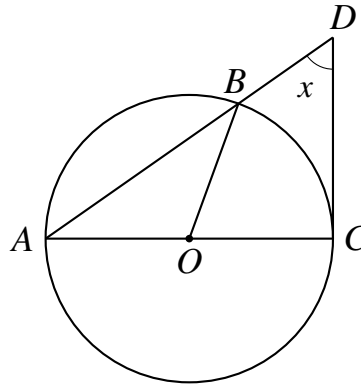
So $ABCD$ is not a cyclic quadrilateral.

Tip: The angle $x + 35$ is **outside** the quadrilateral. Change it to the interior angle $180 - (x + 35) = 145 - x$ before you add the four angles.

Marks: 1 mark for angle $BCD = 145 - x$. 1 mark for $x = 35$. 1 mark for working out a pair of opposite angles. 1 mark for $195 \neq 180$ (or $165 \neq 180$) with a conclusion.

7 (4 marks)

A , B and C are points on a circle, centre O .



Not drawn accurately

AC is a diameter of the circle.

CD is a tangent to the circle at C .

ABD is a straight line. Angle $ADC = x$

Prove that angle $BOC = 180^\circ - 2x$

Angle $OCD = 90^\circ$

(A tangent is perpendicular to the radius.)

So angle $ACD = 90^\circ$

In triangle ACD : angle $CAD = 180 - 90 - x = 90 - x$

(Angles in a triangle add up to 180° .)

So angle $BAC = 90 - x$

Angle $BOC = 2 \times \text{angle } BAC = 2(90 - x) = 180 - 2x$

(The angle at the centre is twice the angle at the circumference.)

Tip: Start from what you know and work towards the result. Do not choose a number for x , and do not start from the answer and work backwards.

Marks: 1 mark for angle $ACD = 90^\circ$ with a reason. 1 mark for angle $BAC = 90 - x$. 1 mark for using the angle at the centre. 1 mark for a complete proof with reasons.