

Product Rule for Counting

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

A café offers 3 starters, 5 main courses and 4 desserts.
Ella chooses one starter, one main course and one dessert.
How many different meals could she choose?

Product rule: multiply the number of choices for each part.

$$3 \times 5 \times 4 = 60$$

Answer: 60

Tip: Multiply the numbers of choices, do not add them. $3 + 5 + 4 = 12$ is wrong.

2 (1 mark)

A two-letter code uses the letters A, B, C, D and E.
Letters may be repeated.
How many different codes are possible?
Circle your answer.

Options: 10, 20, 25, 32

First letter: 5 choices.

Second letter: 5 choices (repeats are allowed).

$$5 \times 5 = 25$$

Answer: 25

3 (3 marks)

A password for a tablet is made of four letters.
Each letter is one of A, B, C, D, E or F

- (a) How many different passwords are possible?

Each of the 4 letters can be any of the 6 letters A to F.

$$6 \times 6 \times 6 \times 6 = 1296$$

Answer: 1296

- (b) How many passwords are possible if no letter can be used more than once?

First letter: 6 choices.

Second letter: 5 choices (one letter has been used).

Third letter: 4 choices.

Fourth letter: 3 choices.

$$6 \times 5 \times 4 \times 3 = 360$$

Answer: 360

Tip: When letters cannot be repeated, the number of choices goes down by 1 each time.
Do not try to list all the passwords.

Marks: 1 mark for $6 \times 5 \times 4 \times 3$. 1 mark for 360.

4 (2 marks)

How many three-digit even numbers are there?

First digit: 1 to 9 (it cannot be 0), so 9 choices.

Second digit: 0 to 9, so 10 choices.

Last digit: 0, 2, 4, 6 or 8 (even), so 5 choices.

$$9 \times 10 \times 5 = 450$$

Answer: 450

Tip: Remember that 0 is even, so there are 5 choices for the last digit. A three-digit number cannot start with 0, so there are 9 choices for the first digit.

Marks: 1 mark for $9 \times 10 \times 5$. 1 mark for 450.

5 (2 marks)

In how many different orders can the five letters of the word MATHS be arranged?

All five letters are different.

First letter: 5 choices, then 4, then 3, then 2, then 1.

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

Answer: 120

Marks: 1 mark for $5 \times 4 \times 3 \times 2 \times 1$. 1 mark for 120.

6 (2 marks)

A locker label is one letter followed by two digits.

The letter is a vowel: A, E, I, O or U

The two digits are different and neither of them is 0

How many different locker labels are possible?

Letter: 5 choices (the vowels).

First digit: 1 to 9, so 9 choices.

Second digit: 8 choices (not 0 and not the digit already used).

$$5 \times 9 \times 8 = 360$$

Answer: 360

Tip: Read the conditions carefully: 0 is not allowed, so there are only 9 choices for the first digit and 8 for the second.

Marks: 1 mark for $5 \times 9 \times 8$ (or equivalent). 1 mark for 360.

7 (4 marks)

- (a) A captain and a vice-captain are chosen from 6 players.
How many different ways can they be chosen?

Captain: 6 choices.

Vice-captain: 5 choices (anyone except the captain).

$$6 \times 5 = 30$$

Answer: 30

Marks: 1 mark for 6×5 . 1 mark for 30.

- (b) Two of the 6 players are chosen to go to a meeting.
How many different pairs of players are there?

$6 \times 5 = 30$ counts each pair twice.

For example, (Amy, Ben) and (Ben, Amy) are the same pair.

So divide by 2: $30 \div 2 = 15$

Answer: 15

Tip: In part (a) the order matters (captain is different from vice-captain). Here it does not, so halve the answer.

Marks: 1 mark for $6 \times 5 \div 2$. 1 mark for 15.

8 (2 marks)

A smoothie bar makes each smoothie from one fruit, one base and one topping.

There are 5 bases.

The number of fruits is the same as the number of toppings.

There are 80 different smoothies possible.

How many toppings are there?

Let t be the number of toppings. Then there are also t fruits.

Product rule: $t \times 5 \times t = 80$

$$5t^2 = 80, \text{ so } t^2 = 16$$

$$t = 4$$

Answer: 4

Tip: $80 \div 5 = 16$ is the number of fruit and topping pairs, not the answer. The fruits and toppings are equal numbers that multiply to 16, so each is $\sqrt{16} = 4$.

Marks: 1 mark for $5t^2 = 80$ or $80 \div 5 = 16$. 1 mark for 4.

9 (3 marks)

Kim says,

“There are more ways to arrange the letters of the word PRISM than there are three-digit codes that use the digits 1 to 7 with no digit repeated.”

Is Kim correct?

You must show your working.

Tick one box.

PRISM has 5 different letters.

Arrangements: $5 \times 4 \times 3 \times 2 \times 1 = 120$

Codes: 7 choices, then 6, then 5 (no repeats).

$$7 \times 6 \times 5 = 210$$

120 is **less** than 210, so Kim is not correct.

Answer: No (120 arrangements, 210 codes)

Marks: 1 mark for $5 \times 4 \times 3 \times 2 \times 1$ or 120. 1 mark for $7 \times 6 \times 5$ or 210. 1 mark for No with 120 and 210.
