

Probability - Sample Spaces and Two-Way Tables - Higher

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Two ordinary fair dice are rolled.
The two scores are multiplied.

(a) Work out the probability that the product is an odd number.

There are $6 \times 6 = 36$ equally likely outcomes.

A product is odd only when **both** scores are odd.

Each dice has 3 odd numbers (1, 3, 5), so there are $3 \times 3 = 9$ such outcomes.

Answer: $\frac{9}{36}$

(b) Work out the probability that the product is 20 or more.

Products of 20 or more:

4×5 , 5×4 (20)

4×6 , 6×4 (24)

5×5 (25), 5×6 , 6×5 (30), 6×6 (36)

That is 8 of the 36 outcomes.

Answer: $\frac{8}{36}$

2 (6 marks)

The table shows the language studied by 120 students.

	French	Spanish	German	Total
Year 10	20	21	19	60
Year 11	20	24	16	60
Total	40	45	35	120

- (a) Work out the probability that a student chosen at random studies French.

40 of the 120 students study French.

Answer: $\frac{40}{120}$

- (b) A student who studies Spanish is chosen at random.
Work out the probability that the student is in Year 11.

The student is chosen from the Spanish students only.

$$\text{Spanish total} = 21 + 24 = 45$$

24 of them are in Year 11.

Answer: $\frac{24}{45}$

Tip: “A student who studies Spanish” means you only choose from the 45 Spanish students, not all 120.

Marks: 1 mark for 45 as the denominator. 1 mark for $\frac{24}{45}$.

- (c) Two different students are chosen at random from the Year 10 students.
Work out the probability that both study German.

There are 60 Year 10 students, and 19 of them study German.

$$\text{First student: } P(\text{German}) = \frac{19}{60}$$

Two **different** students, so one German student has gone.

Second student: $P(\text{German}) = \frac{18}{59}$

$$\frac{19}{60} \times \frac{18}{59} = \frac{342}{3540}$$

Answer: $\frac{342}{3540}$

Tip: The same student cannot be chosen twice, so the second fraction is $\frac{18}{59}$, not $\frac{19}{60}$ again.

Marks: 1 mark for $\frac{19}{60}$. 1 mark for $\times \frac{18}{59}$. 1 mark for $\frac{342}{3540}$ or an equivalent (e.g. $\frac{57}{590}$).

3 (3 marks)

Two fair spinners are each numbered 1 to 5

Both are spun and the numbers are multiplied.

Work out the probability that the product is a multiple of 4

There are $5 \times 5 = 25$ equally likely outcomes.

A product is a multiple of 4 if one spinner shows 4, or both show 2

With a 4: (1,4), (2,4), (3,4), (4,4), (5,4), (4,1), (4,2), (4,3), (4,5)

Both 2: (2,2)

That is 10 of the 25 outcomes.

Answer: $\frac{10}{25}$

Marks: 1 mark for 25 outcomes. 1 mark for a correct list or sample space. 1 mark for $\frac{10}{25}$ or an equivalent.

4 (4 marks)

A holiday club runs one activity in the morning and one activity in the afternoon.
There are 5 morning activities and 6 afternoon activities.
Swimming is one of the morning activities and also one of the afternoon activities.
Each child chooses one morning activity and one afternoon activity.
A child cannot choose swimming for both.

- (a) How many different choices of one morning activity and one afternoon activity are there?

Without the swimming rule: $5 \times 6 = 30$ choices.

Swimming in the morning **and** swimming in the afternoon is only one of these choices.

It is not allowed, so take it away: $30 - 1 = 29$

Answer: 29

Tip: Multiply the numbers of choices first, then remove the one choice that is not allowed.

Marks: 1 mark for 5×6 . 1 mark for 29.

- (b) 2 of the morning activities and 4 of the afternoon activities take place outdoors.
Swimming takes place indoors.
Zainab picks one of the possible choices at random.
Work out the probability that both of her activities take place outdoors.

Choices with both activities outdoors: $2 \times 4 = 8$

None of these includes swimming, so all 8 are allowed.

All 29 choices are equally likely.

$$P(\text{both outdoors}) = \frac{8}{29}$$

Answer: $\frac{8}{29}$

Tip: Zainab picks from the 29 allowed choices, so $\frac{2}{5} \times \frac{4}{6}$ (which uses all 30) is not correct here.

Marks: 1 mark for 8 choices, or a fraction with 29 on the bottom. 1 mark for $\frac{8}{29}$.

5 (5 marks)

25 members of a running club were asked whether they prefer running on roads or on trails.

The table shows the results.

	Roads	Trails
Under 40	5	9
40 and over	8	3

- (a) A member who prefers trails is chosen at random.
Work out the probability that this member is under 40

The member is chosen from those who prefer trails only.

Trails total = $9 + 3 = 12$

9 of them are under 40.

Answer: $\frac{9}{12}$

Tip: "A member who prefers trails" means you choose from the 12 trail runners only, not all 25 members.

Marks: 1 mark for 12 as the denominator. 1 mark for $\frac{9}{12}$ or an equivalent.

- (b) Two different members are chosen at random from the whole club.
Work out the probability that one of them prefers roads and the other prefers trails.

Roads total = $5 + 8 = 13$, trails total = $9 + 3 = 12$, club total = 25

Two **different** members, so the second pick is out of 24.

Road then trail: $\frac{13}{25} \times \frac{12}{24} = \frac{156}{600}$

Trail then road: $\frac{12}{25} \times \frac{13}{24} = \frac{156}{600}$

Add: $\frac{156}{600} + \frac{156}{600} = \frac{312}{600}$

Answer: $\frac{312}{600} \left(= \frac{13}{25} \right)$

Tip: There are two orders (road then trail, and trail then road). Include both, and use 24 for the second pick.

Marks: 1 mark for $\frac{13}{25} \times \frac{12}{24}$. 1 mark for adding both orders (or doubling). 1 mark for $\frac{312}{600}$ or an equivalent.

6 (4 marks)

At a school fair there are two games.

Game A: roll two fair six-sided dice. You win if the total is 10 or more.

Game B: spin a fair four-sided spinner, numbered 1 to 4, twice.

You win if the product of the two numbers is exactly 4

Which game gives a better chance of winning?

Tick a box.

Show working to support your answer.

Game A: two dice give $6 \times 6 = 36$ equally likely outcomes.

Total of 10 or more: (4,6), (5,5), (6,4), (5,6), (6,5), (6,6)

$$P(\text{win}) = \frac{6}{36} = \frac{1}{6}$$

Game B: two spins give $4 \times 4 = 16$ equally likely outcomes.

Product exactly 4: (1,4), (4,1), (2,2)

$$P(\text{win}) = \frac{3}{16}$$

Compare: $\frac{1}{6} = \frac{8}{48}$ and $\frac{3}{16} = \frac{9}{48}$

$\frac{9}{48} > \frac{8}{48}$, so Game B is better.

Answer: Game B ($\frac{3}{16} > \frac{1}{6}$)

Tip: (1,4) and (4,1) are different outcomes, so count both. Then compare the two probabilities in the same form (common denominator or decimals).

Marks: 1 mark for 36 outcomes and a correct count for Game A. 1 mark for $\frac{6}{36}$. 1 mark for $\frac{3}{16}$. 1 mark for Game B from correct probabilities.

7 (4 marks)

A bag contains the eight number tiles shown below.



Anouk takes two tiles from the bag at random, without replacement.

Work out the probability that the numbers on her two tiles differ by exactly 1

The first tile is not replaced, so there are $8 \times 7 = 56$ equally likely ordered pairs.

Pairs of tiles that differ by exactly 1:

a 1 and a 2: $2 \times 3 = 6$ pairs

a 2 and a 3: $3 \times 1 = 3$ pairs

a 3 and a 4: $1 \times 2 = 2$ pairs

That is $6 + 3 + 2 = 11$ pairs, and each can be taken in 2 orders: $11 \times 2 = 22$

$$P(\text{differ by 1}) = \frac{22}{56} = \frac{11}{28}$$

(Or with probabilities: $2 \times \left(\frac{2}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{1}{7} + \frac{1}{8} \times \frac{2}{7} \right) = \frac{22}{56}$)

Answer: $\frac{22}{56} \left(= \frac{11}{28} \right)$

Tip: The first tile is not put back, so the second pick is out of 7. Remember that each pair can be taken in either order.

Marks: 1 mark for 56 ordered pairs (or 28 pairs), or a correct product such as $\frac{2}{8} \times \frac{3}{7}$. 1 mark for identifying the three cases 1-2, 2-3 and 3-4. 1 mark for 22 (or 11) or a correct sum. 1 mark for $\frac{22}{56}$ or an equivalent.
