

# Probability Equations

## Solutions

For AQA GCSE Mathematics (8300) • Higher tier

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Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

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**1** (3 marks)

A bag contains  $n$  counters.  
5 of the counters are green.

- (a) Write down an expression, in terms of  $n$ , for the probability that a counter taken at random is green.

There are  $n$  counters altogether.

5 of them are green.

$$\text{Probability} = \frac{\text{number of green}}{\text{total number}}$$

|                              |
|------------------------------|
| <b>Answer:</b> $\frac{5}{n}$ |
|------------------------------|

- (b) Two counters are taken at random without replacement.  
Write down an expression, in terms of  $n$ , for the probability that both counters are green.  
Give your answer as a single fraction.

First counter green:  $\frac{5}{n}$

The first counter is not put back.

So there are now 4 green counters out of  $n - 1$ .

Second counter green:  $\frac{4}{n - 1}$

Multiply for “and”:

$$\frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)}$$

**Answer:**  $\frac{20}{n(n-1)}$  or  $\frac{20}{n^2-n}$

**Tip:** Without replacement, both the top and the bottom go down by 1 for the second counter:  $\frac{4}{n-1}$ , not  $\frac{4}{n}$ .

*Marks:* 1 mark for  $\frac{5}{n} \times \frac{4}{n-1}$ . 1 mark for the single fraction.

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## 2 (3 marks)

A bag contains only red, blue and green counters.

There are  $x$  red counters,  $2x$  blue counters and 12 green counters.

The probability that a counter taken at random is green is  $\frac{1}{4}$   
Work out the value of  $x$ .

Total number of counters =  $x + 2x + 12$

=  $3x + 12$

$$P(\text{green}) = \frac{12}{3x + 12}$$

This equals  $\frac{1}{4}$ , so  $\frac{12}{3x + 12} = \frac{1}{4}$

Cross-multiply:  $48 = 3x + 12$

Subtract 12:  $3x = 36$

Divide by 3:  $x = 12$

**Answer:** 12

**Tip:** The bottom of the fraction is the **total** number of counters,  $3x + 12$ , not just the red and blue ones.

*Marks:* 1 mark for the total  $3x + 12$ . 1 mark for  $\frac{12}{3x + 12} = \frac{1}{4}$ . 1 mark for  $x = 12$ .

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**3** (4 marks)

- (a) A bag contains only red counters and blue counters.

There are 12 red counters.

A counter is taken at random.

$$P(\text{red}) > 0.3$$

Work out the greatest possible number of blue counters in the bag.

$$\text{Let } b \text{ be the number of blue counters. } P(\text{red}) = \frac{12}{12 + b}$$

$$\frac{12}{12 + b} > 0.3$$

$$12 > 0.3(12 + b) = 3.6 + 0.3b$$

$$8.4 > 0.3b, \text{ so } b < 28$$

$b$  is a whole number less than 28, so the greatest is 27.

Check:  $\frac{12}{39} = 0.307 \dots > 0.3$ , but  $\frac{12}{40} = 0.3$  is not greater than 0.3

**Answer: 27**

**Tip:** The inequality is strict ( $>$ ), so 28 blue counters is not allowed: it gives exactly 0.3

*Marks:* 1 mark for  $\frac{12}{12+b} > 0.3$  or for 40 counters in total. 1 mark for 27.

- (b) A jar contains 40 marbles.

Each marble is green or white.

A marble is taken at random.

$$0.35 < P(\text{green}) < 0.4$$

Work out the number of green marbles in the jar.

$$0.35 \times 40 = 14 \text{ and } 0.4 \times 40 = 16$$

So the number of green marbles is more than 14 and less than 16.

The only whole number is 15.

$$\text{Check: } \frac{15}{40} = 0.375 \checkmark$$

**Answer: 15**

*Marks:* 1 mark for 14 or 16. 1 mark for 15.

**4** (4 marks)

A box contains only milk chocolates and dark chocolates.

number of milk chocolates : number of dark chocolates = 2:3

Priya takes two chocolates at random from the box without replacement.

The probability that she takes one milk chocolate and one dark chocolate is  $\frac{18}{35}$

Work out the number of chocolates in the box.

You must show your working.

milk : dark = 2:3, so let there be  $2k$  milk and  $3k$  dark chocolates,  $5k$  in total.

Milk then dark:  $\frac{2k}{5k} \times \frac{3k}{5k-1}$

Dark then milk:  $\frac{3k}{5k} \times \frac{2k}{5k-1}$

One of each:  $2 \times \frac{6k^2}{5k(5k-1)} = \frac{12k}{5(5k-1)}$

$$\frac{12k}{5(5k-1)} = \frac{18}{35}$$

Cross-multiply:  $420k = 90(5k - 1) = 450k - 90$

$30k = 90$ , so  $k = 3$

Total =  $5k = 15$  chocolates (6 milk, 9 dark)

**Answer:** 15

**Tip:** There are two orders: milk then dark, and dark then milk. Check your answer:

$$\frac{6}{15} \times \frac{9}{14} \times 2 = \frac{108}{210} = \frac{18}{35}$$

**Marks:** 1 mark for  $\frac{2k}{5k} \times \frac{3k}{5k-1}$  (or with 6 and 9 tested). 1 mark for doubling it for the two orders. 1 mark for a correct equation in  $k$  (or a full trial showing  $\frac{18}{35}$ ). 1 mark for 15.

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**5** (5 marks)

A tray has  $n$  cupcakes.

2 of the cupcakes have a coin hidden inside.

Leo takes two cupcakes at random from the tray.

The probability that at least one of Leo's cupcakes has a coin inside is  $\frac{3}{5}$

(a) Show that  $3n^2 - 23n + 30 = 0$

"At least one coin" =  $1 - P(\text{no coins})$

No coins:  $\frac{n-2}{n} \times \frac{n-3}{n-1}$

So  $1 - \frac{(n-2)(n-3)}{n(n-1)} = \frac{3}{5}$ , which gives  $\frac{(n-2)(n-3)}{n(n-1)} = \frac{2}{5}$

Cross-multiply:  $5(n-2)(n-3) = 2n(n-1)$

$$5n^2 - 25n + 30 = 2n^2 - 2n$$

$$3n^2 - 23n + 30 = 0$$

**Answer:**  $3n^2 - 23n + 30 = 0$  shown

**Tip:** Working with “no coins” is much quicker than adding the three ways of getting at least one coin.

*Marks:* 1 mark for  $\frac{n-2}{n} \times \frac{n-3}{n-1}$ . 1 mark for a correct equation without fractions. 1 mark for fully correct working to the given equation.

(b) Work out the value of  $n$ .

Factorise:  $(3n - 5)(n - 6) = 0$

$$n = \frac{5}{3} \text{ or } n = 6$$

$n$  must be a whole number, so  $n = 6$ .

$$\text{Check: } 1 - \frac{4}{6} \times \frac{3}{5} = 1 - \frac{12}{30} = \frac{3}{5} \quad \checkmark$$

**Answer:** 6

*Marks:* 1 mark for  $(3n - 5)(n - 6)$  or the formula used correctly. 1 mark for 6 only.

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## 6 (5 marks)

A pencil case contains 4 red pens and 6 black pens.

Omar puts  $x$  more black pens in the pencil case.

Two pens are then taken at random without replacement.

The probability that both pens are red is  $\frac{2}{35}$

Work out the value of  $x$ .

There are now  $10 + x$  pens, and still 4 red pens.

$$P(\text{both red}) = \frac{4}{10+x} \times \frac{3}{9+x} = \frac{12}{(10+x)(9+x)}$$

$$\frac{12}{(10+x)(9+x)} = \frac{2}{35}$$

Cross-multiply:  $420 = 2(10 + x)(9 + x)$ , so  $(10 + x)(9 + x) = 210$

Expand:  $x^2 + 19x + 90 = 210$

$$x^2 + 19x - 120 = 0$$

Factorise:  $(x + 24)(x - 5) = 0$

$x = 5$  (as  $x$  cannot be negative)

**Answer: 5**

**Tip:** The 4 red pens do not change, but the total does: it becomes  $10 + x$ , then  $9 + x$  for the second pen.

*Marks:* 1 mark for  $\frac{4}{10+x} \times \frac{3}{9+x}$ . 1 mark for  $(10 + x)(9 + x) = 210$ . 1 mark for  $x^2 + 19x - 120 = 0$ . 1 mark for  $(x + 24)(x - 5)$  or the formula. 1 mark for 5.

## 7 (5 marks)

A jar contains  $n$  sweets.

5 of the sweets are lemon flavour.

Tia takes two sweets at random from the jar without replacement.

The probability that exactly one of the two sweets is lemon flavour is  $\frac{35}{66}$

Work out the value of  $n$ .

You must show your working.

Lemon then not lemon:  $\frac{5}{n} \times \frac{n-5}{n-1}$

Not lemon then lemon:  $\frac{n-5}{n} \times \frac{5}{n-1}$

Exactly one:  $\frac{10(n-5)}{n(n-1)} = \frac{35}{66}$

Cross-multiply:  $660(n - 5) = 35n(n - 1)$

$$660n - 3300 = 35n^2 - 35n$$

$$35n^2 - 695n + 3300 = 0$$

Divide by 5:  $7n^2 - 139n + 660 = 0$

Factorise:  $(n - 12)(7n - 55) = 0$

$$n = 12 \text{ or } n = \frac{55}{7}$$

$n$  must be a whole number, so  $n = 12$ .

**Answer:** 12

**Tip:** “Exactly one” can happen in two orders, so there are two products. Reject the answer that is not a whole number.

*Marks:* 1 mark for  $\frac{5}{n} \times \frac{n-5}{n-1}$ . 1 mark for doubling for the two orders. 1 mark for a correct quadratic equation. 1 mark for a correct method to solve it. 1 mark for 12 only.

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