

Parallel and Perpendicular Lines

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the equation of a line parallel to $y = 4x - 1$

Options: $y = -4x + 1$, $y = \frac{1}{4}x - 1$, $y = 4x + 7$, $y = -\frac{1}{4}x$

Parallel lines have the same gradient.

$y = 4x - 1$ has gradient 4.

$y = 4x + 7$ is the only option with gradient 4.

Answer: $y = 4x + 7$

2 (1 mark)

Write down the equation of the line that is parallel to $2y = 6x - 1$ and passes through the origin.

Divide by 2: $y = 3x - \frac{1}{2}$, so the gradient is 3.

A parallel line has the same gradient: $y = 3x + c$

It passes through the origin (0, 0), so $c = 0$.

Answer: $y = 3x$

Tip: The gradient of $2y = 6x - 1$ is not 6. Make y the subject first by dividing every term by 2.

3 (1 mark)

Write down the gradient of a line perpendicular to $y = \frac{2}{3}x + 5$

The gradient of the line is $\frac{2}{3}$.

A perpendicular gradient is the negative reciprocal:

Flip the fraction and change the sign: $\frac{2}{3} \rightarrow -\frac{3}{2}$

Check: $\frac{2}{3} \times \left(-\frac{3}{2}\right) = -1 \checkmark$

Answer: $-\frac{3}{2}$

Tip: You need both steps: flip the fraction **and** change the sign. $\frac{3}{2}$ and $-\frac{2}{3}$ are both wrong.

4 (2 marks)

Work out the equation of the line that is parallel to $y = 3x - 2$ and passes through (2, 9)

Parallel, so the gradient is 3: $y = 3x + c$

Substitute (2, 9): $9 = 3 \times 2 + c$

$9 = 6 + c$, so $c = 3$

Answer: $y = 3x + 3$

Marks: 1 mark for $y = 3x + c$ with (2, 9) substituted. 1 mark for $y = 3x + 3$.

5 (3 marks)

A straight line is perpendicular to $y = 4x - 5$

It passes through the point (8, 1)

Work out the equation of the line.

The gradient of $y = 4x - 5$ is 4.

Perpendicular gradient = $-\frac{1}{4}$ (negative reciprocal of 4)

$$\text{So } y = -\frac{1}{4}x + c$$

$$\text{Substitute } (8, 1): 1 = -\frac{1}{4} \times 8 + c$$

$$1 = -2 + c, \text{ so } c = 3$$

$$\text{Answer: } y = -\frac{1}{4}x + 3$$

Tip: The perpendicular gradient is $-\frac{1}{4}$, not -4 or $\frac{1}{4}$: flip the number and change the sign.

Marks: 1 mark for the gradient $-\frac{1}{4}$. 1 mark for substituting $(8, 1)$. 1 mark for $y = -\frac{1}{4}x + 3$.

6 (2 marks)

Straight line L_1 has equation $2y = x + 6$

Straight line L_2 has equation $y + 2x = 3$

Are the lines L_1 and L_2 perpendicular?

Tick a box.

You must show your working.

Rearrange each into $y = mx + c$.

$$L_1: y = \frac{1}{2}x + 3, \text{ so the gradient is } \frac{1}{2}$$

$$L_2: y = -2x + 3, \text{ so the gradient is } -2$$

$$\text{Multiply the gradients: } \frac{1}{2} \times (-2) = -1$$

The product is -1 , so the lines are perpendicular.

$$\text{Answer: Yes, because } \frac{1}{2} \times (-2) = -1$$

Tip: Two lines are perpendicular when their gradients multiply to give -1 . Always find both gradients first.

Marks: 1 mark for both gradients. 1 mark for Yes with a reason.

7 (3 marks)

$PQRS$ is a parallelogram.

P is $(1, 2)$, Q is $(5, 4)$ and S is $(-2, 5)$

Work out the equation of the line through S and R .

Give your answer in the form $y = mx + c$

In a parallelogram, opposite sides are parallel, so SR is parallel to PQ .

$$\text{Gradient of } PQ = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$$

$$\text{So } SR \text{ also has gradient } \frac{1}{2}: y = \frac{1}{2}x + c$$

$$\text{Substitute } S(-2, 5): 5 = \frac{1}{2} \times (-2) + c$$

$$5 = -1 + c, \text{ so } c = 6$$

$$\text{Answer: } y = \frac{1}{2}x + 6$$

Tip: You do not need to find R . SR is parallel to PQ , and you already know one point on it, S .

Marks: 1 mark for the gradient $\frac{1}{2}$. 1 mark for substituting $(-2, 5)$. 1 mark for $y = \frac{1}{2}x + 6$.

8 (4 marks)

A is $(1, 2)$ and B is $(7, 6)$

Work out the equation of the perpendicular bisector of AB .

The perpendicular bisector goes through the midpoint of AB at right angles to AB .

$$\text{Midpoint} = \left(\frac{1+7}{2}, \frac{2+6}{2} \right) = (4, 4)$$

$$\text{Gradient of } AB = \frac{6-2}{7-1} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Perpendicular gradient} = -\frac{3}{2}$$

$$\text{So } y = -\frac{3}{2}x + c$$

$$\text{Substitute } (4, 4): 4 = -\frac{3}{2} \times 4 + c$$

$$4 = -6 + c, \text{ so } c = 10$$

Answer: $y = -\frac{3}{2}x + 10$

Marks: 1 mark for the midpoint (4, 4). 1 mark for the perpendicular gradient $-\frac{3}{2}$. 1 mark for substituting the midpoint. 1 mark for the correct equation.

9 (4 marks)

The straight line L has equation $y = 2x - 3$

P is the point $(-3, 6)$

N is the point on L that is closest to P , so the line PN is perpendicular to L .

Work out the coordinates of N .

L has gradient 2, so PN has gradient $-\frac{1}{2}$.

Line PN : $y = -\frac{1}{2}x + c$

Substitute $P(-3, 6)$: $6 = -\frac{1}{2} \times (-3) + c$

$$6 = 1.5 + c, \text{ so } c = 4.5$$

Line PN is $y = -\frac{1}{2}x + 4.5$

N is where PN meets L : $2x - 3 = -\frac{1}{2}x + 4.5$

$$2.5x = 7.5, \text{ so } x = 3$$

$$y = 2 \times 3 - 3 = 3$$

Answer: $(3, 3)$

Tip: Check your point in both equations: $2 \times 3 - 3 = 3$ and $-\frac{1}{2} \times 3 + 4.5 = 3$

Marks: 1 mark for the gradient $-\frac{1}{2}$. 1 mark for the equation of line PN . 1 mark for setting the two equations equal. 1 mark for $(3, 3)$.

10 (4 marks)

A is $(2,1)$, B is $(8,4)$ and C is $(k,10)$

Angle $ABC = 90^\circ$

Work out the value of k .

Angle $ABC = 90^\circ$, so BC is perpendicular to AB .

$$\text{Gradient of } AB = \frac{4-1}{8-2} = \frac{3}{6} = \frac{1}{2}$$

So the gradient of BC is -2 .

$$\text{Gradient of } BC = \frac{10-4}{k-8} = \frac{6}{k-8}$$

$$\frac{6}{k-8} = -2$$

$$6 = -2(k-8) = -2k + 16$$

$$2k = 10, \text{ so } k = 5$$

Answer: 5

Tip: The right angle is at B , so the two perpendicular lines are AB and BC . Both go through B .

Marks: 1 mark for $\frac{1}{2}$. 1 mark for -2 . 1 mark for an equation for k . 1 mark for 5.
