

Operations with Algebraic Fractions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Simplify $\frac{2}{x} + \frac{5}{x}$
Circle your answer.

Options: $\frac{7}{x}$, $\frac{7}{2x}$, $\frac{10}{x}$, $\frac{7}{x^2}$

Both fractions have the same denominator, x .

So add the numerators: $2 + 5 = 7$

Keep the denominator the same.

Answer: $\frac{7}{x}$

Tip: Never add the denominators. $\frac{7}{2x}$ is a common wrong answer.

2 (4 marks)

Write as a single fraction in its simplest form.

(a) $\frac{x}{3} + \frac{x+2}{5}$

The common denominator is $3 \times 5 = 15$.

$$\frac{x}{3} = \frac{5x}{15}$$

$$\frac{x+2}{5} = \frac{3(x+2)}{15} = \frac{3x+6}{15}$$

Add the numerators: $5x + 3x + 6 = 8x + 6$

Answer: $\frac{8x + 6}{15}$

Marks: 1 mark for using the common denominator 15. 1 mark for $\frac{8x + 6}{15}$.

(b) $\frac{5}{a} - \frac{7}{3a}$

The common denominator is $3a$.

$$\frac{5}{a} = \frac{15}{3a}$$

Subtract the numerators: $\frac{15}{3a} - \frac{7}{3a} = \frac{8}{3a}$

Answer: $\frac{8}{3a}$

Tip: Don't subtract the denominators. Write both fractions over $3a$, then subtract the numerators only.

Marks: 1 mark for using the common denominator $3a$. 1 mark for $\frac{8}{3a}$.

3 (2 marks)

Simplify $\frac{9a^2}{10} \div \frac{3a}{4}$

Give your answer as a single fraction in its simplest form.

To divide, flip the second fraction and multiply:

$$\frac{9a^2}{10} \times \frac{4}{3a}$$

Multiply the numerators: $9a^2 \times 4 = 36a^2$

Multiply the denominators: $10 \times 3a = 30a$

This gives $\frac{36a^2}{30a}$

Divide the top and bottom by $6a$: $\frac{6a}{5}$

Answer: $\frac{6a}{5}$

Tip: Only flip the **second** fraction when you divide. Then cancel the a as well as the numbers.

Marks: 1 mark for $\frac{9a^2}{10} \times \frac{4}{3a}$ (or $\frac{36a^2}{30a}$). 1 mark for $\frac{6a}{5}$.

4 (3 marks)

Write as a single fraction in its simplest form $\frac{3}{x+1} - \frac{2}{x-2}$

The common denominator is $(x+1)(x-2)$.

$$\frac{3}{x+1} = \frac{3(x-2)}{(x+1)(x-2)}$$

$$\frac{2}{x-2} = \frac{2(x+1)}{(x+1)(x-2)}$$

Subtract the numerators:

$$3(x-2) - 2(x+1)$$

$$= 3x - 6 - 2x - 2$$

$$= x - 8$$

You can leave the denominator in brackets.

Answer: $\frac{x-8}{(x+1)(x-2)}$

Marks: 1 mark for the common denominator $(x+1)(x-2)$. 1 mark for the numerator $3(x-2) - 2(x+1)$. 1 mark for the correct answer.

5 (3 marks)

Simplify fully $\frac{x^2 - 25}{3x} \times \frac{6x^2}{x + 5}$

Factorise $x^2 - 25$ (difference of two squares): $(x + 5)(x - 5)$

$$\frac{(x + 5)(x - 5)}{3x} \times \frac{6x^2}{x + 5}$$

Cancel the common factor $(x + 5)$.

Cancel $\frac{6x^2}{3x} = 2x$

This leaves $2x(x - 5)$.

Answer: $2x(x - 5)$

Tip: When you multiply fractions, don't cross-multiply as if it were an equation. Factorise, cancel, then multiply what is left.

Marks: 1 mark for factorising $x^2 - 25$. 1 mark for cancelling correctly. 1 mark for $2x(x - 5)$ or $2x^2 - 10x$.

6 (4 marks)

Simplify fully $\frac{x + 4}{x^2 - 2x} \div \frac{x^2 + x - 12}{x^2}$

To divide by a fraction, flip it and multiply:

$$\frac{x + 4}{x^2 - 2x} \times \frac{x^2}{x^2 + x - 12}$$

Factorise: $x^2 - 2x = x(x - 2)$

Factorise: $x^2 + x - 12 = (x + 4)(x - 3)$

$$\frac{x + 4}{x(x - 2)} \times \frac{x^2}{(x + 4)(x - 3)}$$

Cancel $(x + 4)$, and cancel x into x^2 .

This leaves $\frac{x}{(x-2)(x-3)}$

Answer: $\frac{x}{(x-2)(x-3)}$

Marks: 1 mark for flipping the second fraction and multiplying. 1 mark for factorising correctly.
1 mark for cancelling correctly. 1 mark for the correct answer.

7 (3 marks)

Show that, for $x \neq 0$

$$\frac{x+7}{4x} - \frac{x-2}{6x} = \frac{x+25}{12x}$$

The lowest common denominator of $4x$ and $6x$ is $12x$.

$$\frac{x+7}{4x} = \frac{3(x+7)}{12x} \quad \text{and} \quad \frac{x-2}{6x} = \frac{2(x-2)}{12x}$$

So the left-hand side is $\frac{3(x+7) - 2(x-2)}{12x}$

Expand the top: $3x + 21 - 2x + 4$

$$= x + 25$$

So the left-hand side is $\frac{x+25}{12x}$, as required.

Answer: $\frac{3(x+7) - 2(x-2)}{12x} = \frac{x+25}{12x}$

Tip: Put the second numerator in a bracket before you subtract: $-2(x-2) = -2x + 4$, not $-2x - 4$.

Marks: 1 mark for a common denominator of $12x$ (or $24x^2$). 1 mark for $3(x+7) - 2(x-2)$. 1 mark for a full correct method to $\frac{x+25}{12x}$.

8 (4 marks)

Simplify fully $(x^2 + 4x) \times \frac{x^2 - 5x + 6}{x^2 + 2x - 8}$

Factorise each part:

$$x^2 + 4x = x(x + 4)$$

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

$$x^2 + 2x - 8 = (x + 4)(x - 2)$$

$$x(x + 4) \times \frac{(x - 2)(x - 3)}{(x + 4)(x - 2)}$$

Cancel $(x + 4)$ and $(x - 2)$.

This leaves $x(x - 3)$.

Answer: $x(x - 3)$ or $x^2 - 3x$

Tip: Factorise every part, including $x^2 + 4x$. Multiplying everything out makes it almost impossible to cancel.

Marks: 1 mark for $x(x + 4)$. 1 mark for $(x - 2)(x - 3)$. 1 mark for $(x + 4)(x - 2)$. 1 mark for the correct answer.

9 (3 marks)

Show that $\frac{x - 3}{x - 1} + \frac{x + 3}{x + 1}$
simplifies to $\frac{ax^2 - b}{x^2 - 1}$ where a and b are integers.

The common denominator is $(x - 1)(x + 1) = x^2 - 1$.

Numerator: $(x - 3)(x + 1) + (x + 3)(x - 1)$

$$(x - 3)(x + 1) = x^2 + x - 3x - 3 = x^2 - 2x - 3$$

$$(x + 3)(x - 1) = x^2 - x + 3x - 3 = x^2 + 2x - 3$$

$$\text{Add: } x^2 - 2x - 3 + x^2 + 2x - 3 = 2x^2 - 6$$

So the answer is $\frac{2x^2 - 6}{x^2 - 1}$, with $a = 2$ and $b = 6$.

Answer: $\frac{2x^2 - 6}{x^2 - 1}$

Tip: In a “show that” question, write out every expansion. Sign errors in the brackets are the most common mistake.

Marks: 1 mark for the common denominator. 1 mark for expanding both brackets on the top correctly. 1 mark for $\frac{2x^2 - 6}{x^2 - 1}$.

10 (3 marks)

Simplify fully $\frac{x^3 + 2x}{6} \times \frac{9}{5x^2 + 10}$

You must show your working.

Factorise: $x^3 + 2x = x(x^2 + 2)$

Factorise: $5x^2 + 10 = 5(x^2 + 2)$

$$\frac{x(x^2 + 2)}{6} \times \frac{9}{5(x^2 + 2)}$$

Cancel $(x^2 + 2)$:

$$\frac{9x}{30}$$

$$= \frac{3x}{10}$$

Answer: $\frac{3x}{10}$

Tip: You can only cancel a whole bracket that is a factor of the top and the bottom. You cannot cancel the x^2 terms on their own.

Marks: 1 mark for $x(x^2 + 2)$ or $5(x^2 + 2)$. 1 mark for cancelling $(x^2 + 2)$ correctly. 1 mark for $\frac{3x}{10}$.
