

nth Term of Quadratic Sequences

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here are the first four terms of a sequence.

2 5 10 17

Circle the expression for the n th term of the sequence.

Options: $3n - 1$, $n^2 + 1$, $n^2 + 2$, $2n^2$

The square numbers n^2 are 1, 4, 9, 16

Each term of the sequence is 1 more: 2, 5, 10, 17

Answer: $n^2 + 1$

2 (2 marks)

Here are the first four terms of a quadratic sequence.

31 29 25 19

Work out the 6th term of the sequence.

First differences: -2 , -4 , -6

Second difference: -2 (the first differences go down by 2 each time)

Next two first differences: -8 and -10

5th term = $19 - 8 = 11$

$$6\text{th term} = 11 - 10 = 1$$

Answer: 1

Tip: The question asks for the 6th term, not the next term. Keep going until you reach the 6th term.

Marks: 1 mark for the second difference -2 or for the 5th term 11. 1 mark for 1.

3 (2 marks)

Work out an expression for the n th term of the sequence

3 6 11 18 27

First differences: 3, 5, 7, 9

Second difference: 2

Half of 2 is 1, so the sequence starts with $1n^2 = n^2$.

Subtract n^2 from each term:

$$3 - 1 = 2, \quad 6 - 4 = 2, \quad 11 - 9 = 2, \quad 18 - 16 = 2$$

What is left is always 2.

Answer: $n^2 + 2$

Marks: 1 mark for n^2 . 1 mark for $n^2 + 2$.

4 (3 marks)

Work out an expression for the n th term of the sequence

5 10 19 32 49

First differences: 5, 9, 13, 17

Second difference: 4

Half of 4 is 2, so the sequence starts with $2n^2$.

$2n^2$ gives 2, 8, 18, 32, 50

Subtract these from the sequence:

$$5 - 2 = 3, 10 - 8 = 2, 19 - 18 = 1, 32 - 32 = 0, 49 - 50 = -1$$

3, 2, 1, 0, -1 is a linear sequence going down by 1, so it is $-n + 4$.

Add the two parts together.

Answer: $2n^2 - n + 4$

Tip: $2n^2$ means $2 \times n^2$, so when $n = 3$ it is $2 \times 9 = 18$, not $(2 \times 3)^2 = 36$.

Marks: 1 mark for $2n^2$. 1 mark for finding the linear part $-n + 4$. 1 mark for the correct answer.

5 (3 marks)

Here are the first four terms of a quadratic sequence.

$$10 \quad 8 \quad 4 \quad -2$$

Work out an expression for the n th term.

First differences: -2, -4, -6

Second difference: -2

Half of -2 is -1, so the sequence starts with $-n^2$.

$-n^2$ gives -1, -4, -9, -16

Subtract these from the sequence (which means adding n^2):

$$10 + 1 = 11, 8 + 4 = 12, 4 + 9 = 13, -2 + 16 = 14$$

11, 12, 13, 14 is the linear sequence $n + 10$.

Check with $n = 3$: $-9 + 3 + 10 = 4$ ✓

Answer: $-n^2 + n + 10$

Tip: Always work out differences as (next term) – (term before). Here $8 - 10 = -2$, so the differences are negative.

Marks: 1 mark for $-n^2$. 1 mark for finding the linear part $n + 10$. 1 mark for the correct answer.

6 (2 marks)

The n th term of a sequence is $n^2 + 4n$

Is 140 a term in the sequence? Tick a box.

You must show your working.

Set the n th term equal to 140: $n^2 + 4n = 140$

Rearrange: $n^2 + 4n - 140 = 0$

Factorise: $(n + 14)(n - 10) = 0$

So $n = 10$ or $n = -14$

$n = 10$ is a positive whole number, so 140 is a term.

Check: $10^2 + 4 \times 10 = 100 + 40 = 140$ ✓

Answer: Yes. It is the 10th term.

Tip: A number is in the sequence only if n comes out as a positive **whole** number.

Marks: 1 mark for forming and solving $n^2 + 4n = 140$. 1 mark for Yes with $n = 10$.

7 (4 marks)

Three facts about a quadratic sequence are given below.

The 1st term is 5

The 4th term is 47

The second difference is 6

Work out an expression for the n th term of the sequence.

Second difference 6, so the n^2 part is $\frac{6}{2}n^2 = 3n^2$.

So the n th term is $3n^2 + bn + c$

1st term ($n = 1$): $3 + b + c = 5$, so $b + c = 2$

4th term ($n = 4$): $48 + 4b + c = 47$, so $4b + c = -1$

Subtract the first equation from the second: $3b = -3$, so $b = -1$

Substitute into $b + c = 2$: $c = 3$

Check: the sequence 5, 13, 27, 47 has differences 8, 14, 20, which go up by 6 ✓

Answer: $3n^2 - n + 3$

Tip: Use $n = 4$ for the 4th term: $3 \times 4^2 = 48$. A common slip is to work out $(3 \times 4)^2$ instead.

Marks: 1 mark for $3n^2$. 1 mark for an equation in b and c . 1 mark for solving to find b or c . 1 mark for the correct answer.

8 (3 marks)

The n th term of a sequence is $n^2 + 5n + 6$

Show that every term of the sequence is an even number.

Factorise: $n^2 + 5n + 6 = (n + 2)(n + 3)$

$n + 2$ and $n + 3$ are consecutive whole numbers (one follows straight after the other).

Of any two consecutive whole numbers, one is even.

An even number multiplied by any whole number is even.

So $(n + 2)(n + 3)$ is always even.

For example, the first term is $3 \times 4 = 12$ ✓

Answer: $(n + 2)(n + 3)$ is the product of two consecutive whole numbers, one of which is even, so every term is even.

Tip: Showing that the first few terms are even is not enough. You need a reason that works for every value of n .

Marks: 1 mark for $(n + 2)(n + 3)$. 1 mark for saying $n + 2$ and $n + 3$ are consecutive. 1 mark for a full explanation that one of them is even, so the product is even.

9 (3 marks)

The n th term of a sequence is $n^2 - 24n + 155$

By completing the square, show that all the terms of the sequence have two or more digits.

Halve -24 to get -12 : $n^2 - 24n = (n - 12)^2 - 144$

So $n^2 - 24n + 155 = (n - 12)^2 - 144 + 155$

$= (n - 12)^2 + 11$

A square is never negative, so $(n - 12)^2 \geq 0$.

So every term is at least $0 + 11 = 11$.

11 has two digits, so every term has two or more digits.

Answer: $(n - 12)^2 + 11 \geq 11$, so every term has at least two digits.

Marks: 1 mark for $(n - 12)^2$. 1 mark for $(n - 12)^2 + 11$. 1 mark for saying the smallest value is 11, so every term has two or more digits.
