

Negative and Fractional Indices

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (7 marks)

Work out the value of

(a) 5^{-2}

A negative power means “one over”: $a^{-n} = \frac{1}{a^n}$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Answer: $\frac{1}{25}$

Tip: A negative power does not make the answer negative. 5^{-2} is not -25 or -10 .

(b) $64^{\frac{1}{2}}$

A power of $\frac{1}{2}$ means square root.

$$64^{\frac{1}{2}} = \sqrt{64} = 8$$

Answer: 8

(c) $27^{\frac{2}{3}}$

The bottom of the fraction (3) means cube root.

The top (2) means square.

$$\sqrt[3]{27} = 3$$

$$3^2 = 9$$

Answer: 9

Tip: Do not multiply by the power: $27 \times \frac{2}{3} = 18$ is wrong. Take the root first, then the power.

(d) $16^{-\frac{3}{4}}$

The minus sign means “one over”: $16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$

Now work out $16^{\frac{3}{4}}$:

Fourth root: $\sqrt[4]{16} = 2$ (because $2 \times 2 \times 2 \times 2 = 16$)

Cube: $2^3 = 8$

So $16^{-\frac{3}{4}} = \frac{1}{8}$

Answer: $\frac{1}{8}$

Tip: The minus sign tells you to take the reciprocal of the answer. It does not turn the power upside down ($16^{\frac{4}{3}}$ is wrong).

Marks: 1 mark for 2^3 or $\frac{1}{16^{\frac{3}{4}}}$. 1 mark for $\frac{1}{8}$.

(e) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

The minus sign means flip the fraction:

$$\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}}$$

Cube root: $\sqrt[3]{\frac{64}{27}} = \frac{4}{3}$

Square: $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Answer: $\frac{16}{9}$

Marks: 1 mark for $\frac{64}{27}$ or $\frac{9}{16}$. 1 mark for $\frac{16}{9}$.

2 (1 mark)

Circle the value of $8^{-\frac{1}{3}}$

Options: -2 , $-\frac{8}{3}$, $\frac{1}{2}$, $\frac{1}{24}$

$$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$$

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$\text{So } 8^{-\frac{1}{3}} = \frac{1}{2}$$

Answer: $\frac{1}{2}$

3 (3 marks)

Work out the value of $\left(\frac{2}{5}\right)^{-2} - 2^{-3}$

Give your answer as a mixed number.

A negative power means take the reciprocal.

$$\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Use a common denominator of 8: $\frac{25}{4} = \frac{50}{8}$

$$\frac{50}{8} - \frac{1}{8} = \frac{49}{8}$$

As a mixed number: $49 \div 8 = 6$ remainder 1, so $6\frac{1}{8}$

Answer: $6\frac{1}{8}$

Tip: 2^{-3} is not -6 or -8 . A negative power gives a reciprocal: $2^{-3} = \frac{1}{8}$.

Marks: 1 mark for $\frac{25}{4}$. 1 mark for $\frac{1}{8}$. 1 mark for $6\frac{1}{8}$.

4 (5 marks)

Find the value of x .

(a) $2^x = \frac{1}{32}$

$$32 = 2^5$$

$$\text{So } \frac{1}{32} = \frac{1}{2^5} = 2^{-5}$$

$$x = -5$$

Answer: -5

(b) $3^x = 9\sqrt{3}$

Write each part as a power of 3:

$$9 = 3^2 \text{ and } \sqrt{3} = 3^{\frac{1}{2}}$$

$$9\sqrt{3} = 3^2 \times 3^{\frac{1}{2}}$$

$$\text{Add the powers: } 3^{2+\frac{1}{2}} = 3^{\frac{5}{2}}$$

$$\text{So } x = \frac{5}{2}$$

$$\text{Answer: } \frac{5}{2}$$

Tip: A square root is a power of $\frac{1}{2}$, so $\sqrt{3} = 3^{\frac{1}{2}}$. Use this to write everything as a power of 3.

Marks: 1 mark for $\sqrt{3} = 3^{\frac{1}{2}}$. 1 mark for $\frac{5}{2}$ (or 2.5).

(c) $16^x = 8$

Write both numbers as powers of 2:

$$16 = 2^4, \text{ so } 16^x = (2^4)^x = 2^{4x}$$

$$8 = 2^3$$

$$\text{So } 2^{4x} = 2^3$$

The powers must be equal: $4x = 3$

$$x = \frac{3}{4}$$

$$\text{Answer: } \frac{3}{4}$$

Marks: 1 mark for $2^{4x} = 2^3$ (or $16^{\frac{1}{4}} = 2$). 1 mark for $\frac{3}{4}$.

5 (3 marks)

Simplify fully

(a) $(x^6)^{\frac{1}{3}}$

For a power of a power, multiply the powers:

$$6 \times \frac{1}{3} = 2$$

$$\text{Answer: } x^2$$

(b) $(16x^8y^{-4})^{\frac{1}{2}}$

Apply the power $\frac{1}{2}$ to each part:

$$16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$(x^8)^{\frac{1}{2}} = x^{8 \times \frac{1}{2}} = x^4$$

$$(y^{-4})^{\frac{1}{2}} = y^{-4 \times \frac{1}{2}} = y^{-2}$$

So the answer is $4x^4y^{-2}$, which is the same as $\frac{4x^4}{y^2}$

Answer: $4x^4y^{-2}$ or $\frac{4x^4}{y^2}$

Marks: 2 marks for the fully correct answer. 1 mark if two of the three parts are correct.

6 (3 marks)

Write $\frac{4^3 \times 2^{-1}}{8^{\frac{2}{3}}}$ as a power of 2

Write each number as a power of 2:

$$4^3 = (2^2)^3 = 2^6$$

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2$$

So the expression is $\frac{2^6 \times 2^{-1}}{2^2}$

Multiplying: add the powers. Dividing: subtract the powers.

$$2^{6+(-1)-2} = 2^3$$

Answer: 2^3

Marks: 1 mark for 2^6 or 2^2 . 1 mark for a correct method to combine the powers. 1 mark for 2^3 .

7 (2 marks)

Write $\frac{1}{\sqrt{100^m}}$ as a power of 10 in terms of m .

$$100 = 10^2, \text{ so } 100^m = (10^2)^m = 10^{2m}$$

A square root is a power of $\frac{1}{2}$: $\sqrt{10^{2m}} = 10^{2m \times \frac{1}{2}} = 10^m$

One over a power gives a negative power:

$$\frac{1}{10^m} = 10^{-m}$$

Answer: 10^{-m}

Tip: Do not forget the “one over” at the end: it makes the power negative.

Marks: 1 mark for 10^{2m} or 10^m (or $100^{\frac{m}{2}}$). 1 mark for 10^{-m} .

8 (3 marks)

Write $\frac{8^{x+1}}{\sqrt{2}}$ as a power of 2 in terms of x .

Write each part as a power of 2:

$$8 = 2^3, \text{ so } 8^{x+1} = (2^3)^{x+1} = 2^{3x+3}$$

$$\sqrt{2} = 2^{\frac{1}{2}}$$

Dividing, so subtract the powers:

$$2^{3x+3-\frac{1}{2}} = 2^{3x+\frac{5}{2}}$$

Answer: $2^{3x+\frac{5}{2}}$

Tip: Multiply every term in the bracket by 3: $3(x+1) = 3x+3$, not $3x+1$.

Marks: 1 mark for 2^{3x+3} (or $2^{3(x+1)}$). 1 mark for $\sqrt{2} = 2^{\frac{1}{2}}$. 1 mark for $2^{3x+\frac{5}{2}}$.
