

Mixed Sine and Cosine Rule Problems

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

For each triangle, tick the rule you would use to work out the missing value.

	Sine rule	Cosine rule
Two sides and the angle between them are known. Work out the third side.		✓
Two angles and one side are known. Work out another side.	✓	
All three sides are known. Work out an angle.		✓

Use the **cosine rule** when you know two sides and the angle between them.

Also use the **cosine rule** when you know all three sides.

Use the **sine rule** when you know a side and the angle opposite it.

With two angles, you can find the third angle, so you always have a side and its opposite angle.

Answer: Cosine rule, Sine rule, Cosine rule

Marks: 2 marks for all three correct. 1 mark for two correct.

2 (4 marks)

A triangle has sides 6 cm and 10 cm with an included angle of 40°
Work out the perimeter of the triangle.

“Included angle” means the angle between the two sides.

So use the cosine rule to find the third side, x .

$$x^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 40^\circ$$

$$x^2 = 136 - 91.9 \dots = 44.07 \dots$$

$$x = \sqrt{44.07 \dots} = 6.638 \dots$$

$$\text{Perimeter} = 6 + 10 + 6.638 \dots = 22.63 \dots$$

Answer: 22.6 cm

Marks: 1 mark for substituting into the cosine rule. 1 mark for the square root. 1 mark for the third side, 6.64. 1 mark for 22.6.

3 (4 marks)

In triangle XYZ , $XY = 9$ cm, angle $X = 42^\circ$ and angle $Y = 73^\circ$
Work out the area of the triangle.

To use $\frac{1}{2}ab \sin C$ we need two sides and the angle between them.

We only know one side, so find another side first.

$$\text{Angle } Z = 180 - 42 - 73 = 65^\circ \text{ (angles in a triangle)}$$

Sine rule: XZ is opposite 73° and $XY = 9$ is opposite 65° .

$$XZ = \frac{9 \sin 73^\circ}{\sin 65^\circ} = 9.496 \dots$$

Angle $X = 42^\circ$ is between XY and XZ .

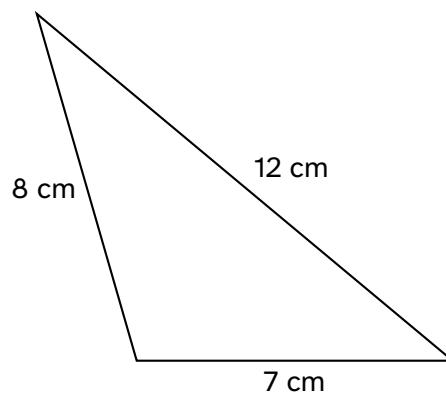
$$\text{Area} = \frac{1}{2} \times 9 \times 9.496 \dots \times \sin 42^\circ = 28.59 \dots$$

Answer: 28.6 cm²

Marks: 1 mark for angle $Z = 65^\circ$. 1 mark for the sine rule. 1 mark for the area method. 1 mark for 28.6.

4 (4 marks)

A triangle has sides 7 cm, 8 cm and 12 cm.



Not drawn accurately

Work out the area of the triangle.

Give your answer to 3 significant figures.

We know three sides, so use the cosine rule to find an angle first.

Find θ , the angle opposite the 12 cm side.

$$\cos \theta = \frac{7^2 + 8^2 - 12^2}{2 \times 7 \times 8} = \frac{-31}{112}$$

$$\theta = \cos^{-1}\left(\frac{-31}{112}\right) = 106.06 \dots^\circ$$

θ is between the 7 cm and 8 cm sides.

$$\text{Area} = \frac{1}{2} \times 7 \times 8 \times \sin 106.06 \dots^\circ = 26.90 \dots$$

Answer: 26.9 cm²

Tip: In $\frac{1}{2}ab \sin C$, the angle must be between sides a and b . Here, 106.06° goes with 7 cm and 8 cm.

Marks: 1 mark for the cosine rule for an angle. 1 mark for the correct angle. 1 mark for $\frac{1}{2}ab \sin C$ with your angle. 1 mark for 26.9.

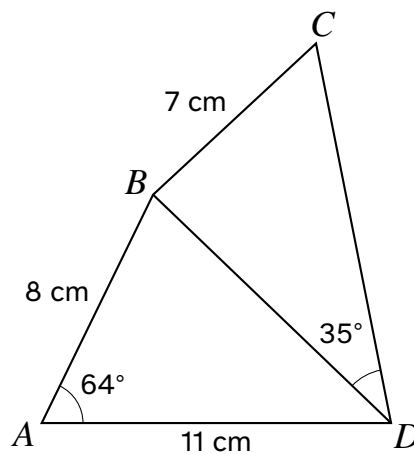
5 (6 marks)

$ABCD$ is a quadrilateral.

$AB = 8$ cm, $AD = 11$ cm, angle $BAD = 64^\circ$

$BC = 7$ cm and angle $BDC = 35^\circ$

Angle BCD is acute.



Not drawn accurately

(a) Work out the length BD .

In triangle ABD we know two sides and the angle between them.

So use the cosine rule.

$$BD^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 64^\circ$$

$$BD^2 = 185 - 77.1 \dots = 107.84 \dots$$

$$BD = \sqrt{107.84 \dots} = 10.38 \dots$$

Answer: 10.4 cm

Marks: 1 mark for substituting into the cosine rule. 1 mark for the square root. 1 mark for 10.4.

- (b) Work out the size of angle BCD .

In triangle BCD we know $BC = 7$ and the angle opposite it, 35° .

So use the sine rule.

BD is opposite angle BCD .

$$\frac{\sin C}{10.38 \dots} = \frac{\sin 35^\circ}{7}$$

$$\sin C = \frac{10.38 \dots \times \sin 35^\circ}{7} = 0.8509 \dots$$

$$C = \sin^{-1}(0.8509 \dots) = 58.31 \dots$$

Angle BCD is acute, so this is the answer.

Answer: 58.3°

Tip: Use the full value of BD from part (a), not 10.4.

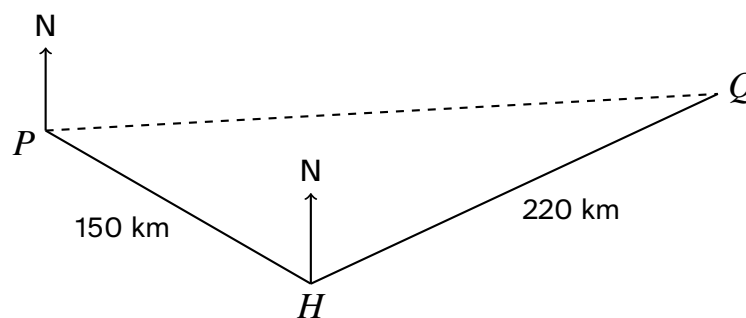
Marks: 1 mark for the sine rule. 1 mark for $\sin C = 0.85 \dots$ 1 mark for 58.3 .

6 (6 marks)

A helicopter is based at H .

Oil platform P is 150 km from H on a bearing of 300°

Oil platform Q is 220 km from H on a bearing of 065°



Not drawn accurately

- (a) Show that the distance from P to Q is 330 km to the nearest km.
You must show your working.

Angle PHQ is the angle turned clockwise from bearing 300° to bearing 065°

$$\text{Angle } PHQ = (360 - 300) + 65 = 125^\circ$$

$$\text{Cosine rule: } PQ^2 = 150^2 + 220^2 - 2 \times 150 \times 220 \times \cos 125^\circ$$

$$PQ^2 = 22\,500 + 48\,400 + 37\,856.2 \dots = 108\,756.2 \dots$$

$$PQ = \sqrt{108\,756.2 \dots} = 329.78 \dots$$

$$329.78 \dots = 330 \text{ km to the nearest km } \checkmark$$

Tip: The angle between the two bearings is not $300 - 65 = 235^\circ$. Draw the North line at H and add the two parts either side of it: $60^\circ + 65^\circ$.

Marks: 1 mark for angle $PHQ = 125^\circ$ used in the cosine rule. 1 mark for seeing 329.8 (more accurate than 330).

- (b) The helicopter flies directly from P to Q .
Work out the bearing of Q from P .

First find angle HPQ with the sine rule.

$HQ = 220$ is opposite angle HPQ , and $PQ = 329.78 \dots$ is opposite 125° .

$$\frac{\sin HPQ}{220} = \frac{\sin 125^\circ}{329.78 \dots}$$

$$\sin HPQ = 0.5464 \dots, \text{ so angle } HPQ = 33.12 \dots^\circ$$

The bearing of H from P is the back bearing of 300° : $300 - 180 = 120^\circ$

Q is anticlockwise from H as seen from P , so subtract.

Bearing of Q from $P = 120 - 33.12... = 86.87...$

Answer: 087°

Tip: Finding angle HPQ is not the end. Use the North line at P to turn it into a bearing, and write bearings with three figures.

Marks: 1 mark for the sine rule or cosine rule to find an angle at P . 1 mark for 33.1° . 1 mark for the bearing of H from P (120°). 1 mark for an answer from 086.8 to 087.
