

Linear Inequalities and Set Notation

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Solve

(a) $4x + 3 \leq 23$

Subtract 3 from both sides:

$$4x \leq 20$$

Divide both sides by 4:

$$x \leq 5$$

Answer: $x \leq 5$

Tip: Keep the inequality sign all the way through. Writing $x = 5$ as your answer loses a mark.

Marks: 1 mark for $4x \leq 20$. 1 mark for $x \leq 5$.

(b) $5x + 6 > 3x + 15$

Subtract $3x$ from both sides:

$$2x + 6 > 15$$

Subtract 6 from both sides:

$$2x > 9$$

Divide both sides by 2:

$$x > 4.5$$

Answer: $x > 4.5$

Marks: 1 mark for $2x > 9$. 1 mark for $x > 4.5$.

2 (3 marks)

Solve $5 - 2x \geq 11$

Give your answer in set notation.

Subtract 5 from both sides:

$$-2x \geq 6$$

Divide both sides by -2 .

Dividing by a negative number reverses the inequality sign:

$$x \leq -3$$

In set notation, $\{x: x \leq -3\}$ means “the set of all x such that $x \leq -3$ ”.

Answer: $\{x: x \leq -3\}$

Tip: Many students forget to reverse the sign when they divide by -2 . You can avoid this: add $2x$ to both sides to get $5 \geq 11 + 2x$, then $-6 \geq 2x$, so $-3 \geq x$.

Marks: 1 mark for $-2x \geq 6$. 1 mark for $x \leq -3$. 1 mark for writing it in set notation.

3 (5 marks)

(a) Solve $-3 \leq 2x + 1 < 9$

Do the same thing to all three parts.

Subtract 1 from all three parts:

$$-4 \leq 2x < 8$$

Divide all three parts by 2:

$$-2 \leq x < 4$$

Answer: $-2 \leq x < 4$

Tip: A common mistake is to subtract 1 from only one side. Whatever you do, do it to all three parts.

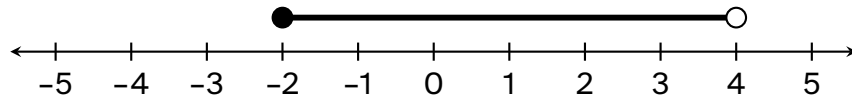
Marks: 1 mark for $-4 \leq 2x < 8$ (or for -2 and 4 seen). 1 mark for $-2 \leq x < 4$.

- (b) Show your answer to part (a) on the number line.

-2 is included ($\leq$), so draw a filled circle at -2 .

4 is not included ($<$), so draw an open circle at 4 .

Join the two circles with a line.



Answer: Filled circle at -2 , open circle at 4 , joined

Marks: 1 mark for the correct circles at -2 and 4 . 1 mark for a line joining them.

- (c) Write down the integers that satisfy the inequality in part (a).

From part (a), $-2 \leq x < 4$.

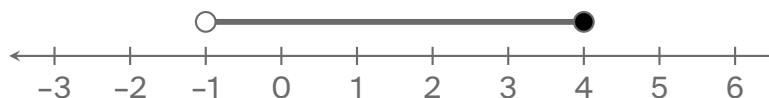
-2 is included, but 4 is not.

Answer: $-2, -1, 0, 1, 2, 3$

Tip: Don't forget 0. It is an integer too.

4 (2 marks)

Here is a number line.



- (a) Write down the inequality shown on the number line.

The circle at -1 is open, so -1 is not included. Use $<$.

The circle at 4 is filled, so 4 is included. Use $\leq$.

Answer: $-1 < x \leq 4$

- (b) Circle the set notation for this inequality.

Options: $\{x: -1 \leq x < 4\}$, $\{x: -1 < x \leq 4\}$, $\{0, 1, 2, 3, 4\}$, $\{-1, 4\}$

$\{x: \dots\}$ means “the set of all x such that ...”

So we need $\{x: -1 < x \leq 4\}$, using the inequality from part (a).

$\{0, 1, 2, 3, 4\}$ only lists whole numbers, but x can be any number in the range, e.g. 2.5.

Answer: $\{x: -1 < x \leq 4\}$

5 (2 marks)

Solve $7 < 2 - \frac{x}{4}$

Subtract 2 from both sides:

$$5 < -\frac{x}{4}$$

Multiply both sides by -4 .

Multiplying by a negative number reverses the sign:

$$-20 > x$$

This is the same as $x < -20$.

Answer: $x < -20$

Marks: 1 mark for $5 < -\frac{x}{4}$ or $28 < 8 - x$. 1 mark for $x < -20$.

6 (3 marks)

Solve $4(x - 3) > 2x + 5$

Give your answer in set notation.

Expand the bracket:

$$4x - 12 > 2x + 5$$

Subtract $2x$ from both sides:

$$2x - 12 > 5$$

Add 12 to both sides:

$$2x > 17$$

Divide both sides by 2:

$$x > 8.5$$

In set notation: $\{x: x > 8.5\}$

Answer: $\{x: x > 8.5\}$

Marks: 1 mark for expanding the bracket correctly. 1 mark for $x > 8.5$. 1 mark for writing your answer in set notation.

7 (2 marks)

Hollie says,

“The inequalities $-4 < 2n \leq 6$ and $-2 \leq n < 3$ are satisfied by exactly the same integers.”

Is Hollie correct?

You must show your working.

Divide all three parts of $-4 < 2n \leq 6$ by 2:

$$-2 < n \leq 3$$

So the integers are $-1, 0, 1, 2, 3$.

The integers that satisfy $-2 \leq n < 3$ are $-2, -1, 0, 1, 2$.

The first includes 3 but not -2 . The second includes -2 but not 3.

Answer: No. The first gives $-1, 0, 1, 2, 3$ but the second gives $-2, -1, 0, 1, 2$

Tip: Look carefully at each end: $<$ means the end value is **not** included, $\leq$ means it is.

Marks: 1 mark for $-2 < n \leq 3$ or for listing either set of integers. 1 mark for No with both lists (or a correct comparison of the end values).

8 (3 marks)

n is an integer.

$$5n - 3 > 2n + 9 \quad \text{and} \quad 2(n + 1) < 17$$

Work out all the possible values of n .

Solve the first inequality:

$$5n - 3 > 2n + 9$$

$$3n > 12$$

$$n > 4$$

Solve the second inequality:

$$2(n + 1) < 17$$

$$2n + 2 < 17$$

$$2n < 15$$

$$n < 7.5$$

$$\text{So } 4 < n < 7.5$$

4 is not included, and the integers below 7.5 go up to 7.

Answer: 5, 6, 7

Tip: $n < 7.5$ still allows $n = 7$. Don't stop at 6.

Marks: 1 mark for $n > 4$. 1 mark for $n < 7.5$. 1 mark for 5, 6, 7.

9 (3 marks)

A pottery studio charges a membership fee of £18, plus £7.50 for each session. Keeley has £100 to spend on membership and sessions. Form and solve an inequality to work out the greatest number of sessions Keeley can pay for.

Let s be the number of sessions.

The total cost must be no more than £100:

$$18 + 7.5s \leq 100$$

Subtract 18 from both sides:

$$7.5s \leq 82$$

Divide both sides by 7.5:

$s \leq 10.93 \dots$ (without a calculator: $7.5 \times 10 = 75$ and $7.5 \times 11 = 82.5$, so s is between 10 and 11)

s must be a whole number, so the greatest number of sessions is 10.

Answer: 10

Tip: Don't round 10.93 up to 11. Keeley cannot afford 11 sessions ($18 + 82.50 = £100.50$).

Marks: 1 mark for $18 + 7.5s \leq 100$ (or $<$). 1 mark for $s \leq 10.9 \dots$. 1 mark for 10.

10 (3 marks)

a and b are integers.

$$-7 < 2a + 1 \leq 5 \quad \text{and} \quad 3 \leq b < 8$$

Work out the smallest possible value of $a^2 - b$

Solve the inequality for a . Subtract 1 from all three parts:

$$-8 < 2a \leq 4$$

Divide all three parts by 2:

$$-4 < a \leq 2, \text{ so } a \text{ can be } -3, -2, -1, 0, 1, 2.$$

To make $a^2 - b$ as small as possible, make a^2 as small as possible and b as large as possible.

The smallest value of a^2 is $0^2 = 0$ (squares are never negative).

The largest value of b is 7 (8 is not included).

$$a^2 - b = 0 - 7 = -7$$

Answer: -7

Tip: Using $a = -3$ is a trap: $(-3)^2 = 9$, which is large, not small.

Marks: 1 mark for $-4 < a \leq 2$ (or listing the values of a). 1 mark for using $a = 0$ and $b = 7$. 1 mark for -7 .
