

Linear Equations with Fractions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Solve

(a) $\frac{x+4}{3} = 5$

Multiply both sides by 3 to clear the fraction:

$$x + 4 = 15$$

Subtract 4 from both sides:

$$x = 11$$

Answer: $x = 11$

Marks: 1 mark for $x + 4 = 15$. 1 mark for the correct answer.

(b) $\frac{2x}{5} - 1 = 7$

Add 1 to both sides:

$$\frac{2x}{5} = 8$$

Multiply both sides by 5:

$$2x = 40$$

Divide both sides by 2:

$$x = 20$$

Answer: $x = 20$

Marks: 1 mark for $\frac{2x}{5} = 8$ or $2x - 5 = 35$. 1 mark for the correct answer.

2 (3 marks)

Solve $\frac{x}{3} + \frac{x}{4} = 14$

The lowest common multiple of 3 and 4 is 12.

Multiply **every** term by 12:

$$\frac{x}{3} \times 12 = 4x$$

$$\frac{x}{4} \times 12 = 3x$$

$$14 \times 12 = 168$$

$$\text{So } 4x + 3x = 168$$

$$7x = 168$$

Divide both sides by 7: $x = 24$

Answer: $x = 24$

Tip: Multiply **every** term by 12, including the 14 on the right-hand side.

Marks: 1 mark for multiplying by 12 (or another common multiple). 1 mark for $7x = 168$. 1 mark for the correct answer.

3 (3 marks)

Solve $\frac{3x - 2}{4} = \frac{x + 5}{3}$

Multiply both sides by 12 (or cross-multiply):

$$3(3x - 2) = 4(x + 5)$$

Expand the brackets:

$$9x - 6 = 4x + 20$$

Subtract $4x$ from both sides and add 6 to both sides:

$$5x = 26$$

Divide both sides by 5: $x = \frac{26}{5}$

Answer: $x = \frac{26}{5}$ (or 5.2)

Tip: Put brackets round the tops of the fractions. $3(3x - 2) = 9x - 6$, but writing $3 \times 3x - 2$ gives $9x - 2$, which is wrong.

Marks: 1 mark for $3(3x - 2) = 4(x + 5)$. 1 mark for $5x = 26$. 1 mark for the correct answer (as a fraction or decimal).

4 (3 marks)

Solve $\frac{x+3}{2} - \frac{x-1}{3} = 4$

The lowest common multiple of 2 and 3 is 6.

Multiply every term by 6:

$$3(x+3) - 2(x-1) = 24$$

Expand the brackets:

$$3x + 9 - 2x + 2 = 24$$

Note that $-2 \times (-1) = +2$.

Simplify: $x + 11 = 24$

Subtract 11: $x = 13$

Answer: $x = 13$

Tip: The minus sign in front of the second fraction applies to the whole top:

$$-2(x-1) = -2x + 2, \text{ not } -2x - 2.$$

Marks: 1 mark for multiplying by 6. 1 mark for a correct expansion, including $-2 \times (-1) = +2$. 1 mark for the correct answer.

5 (4 marks)

Solve $\frac{5x+1}{3} + 2 = \frac{2x-1}{2}$

The lowest common multiple of 3 and 2 is 6.

Multiply every term by 6:

$$2(5x+1) + 12 = 3(2x-1)$$

Expand the brackets:

$$10x + 2 + 12 = 6x - 3$$

Simplify: $10x + 14 = 6x - 3$

Subtract $6x$ from both sides: $4x + 14 = -3$

Subtract 14 from both sides: $4x = -17$

Divide both sides by 4: $x = -\frac{17}{4}$

Answer: $x = -\frac{17}{4}$ (or -4.25)

Tip: Do not forget to multiply the 2 by 6 as well. It becomes 12.

Marks: 1 mark for multiplying every term by 6. 1 mark for expanding. 1 mark for $4x = -17$. 1 mark for the correct answer.

6 (4 marks)

Solve $\frac{3x-1}{4} - \frac{x-5}{6} = \frac{x}{2}$

The lowest common multiple of 4, 6 and 2 is 12.

Multiply every term by 12:

$$3(3x-1) - 2(x-5) = 6x$$

Expand the brackets:

$$9x - 3 - 2x + 10 = 6x$$

Note that $-2 \times (-5) = +10$.

Simplify: $7x + 7 = 6x$

Subtract $6x$ from both sides: $x + 7 = 0$

Subtract 7: $x = -7$

Check: $\frac{-22}{4} - \frac{-12}{6} = -5.5 + 2 = -3.5$ and $\frac{-7}{2} = -3.5$ ✓

Answer: $x = -7$

Tip: The right-hand side must be multiplied by 12 too: $12 \times \frac{x}{2} = 6x$. The minus in front of the second fraction changes the sign of **both** terms on top.

Marks: 1 mark for multiplying every term by 12 (or another common multiple). 1 mark for $9x - 3 - 2x + 10$. 1 mark for $7x + 7 = 6x$. 1 mark for the correct answer.

7 (4 marks)

Beth's age is x years.

Her brother is 4 years older than her.

Half of Beth's age plus a third of her brother's age is 13

Work out Beth's age.

Her brother's age is $x + 4$.

Half of Beth's age is $\frac{x}{2}$.

A third of her brother's age is $\frac{x + 4}{3}$.

So the equation is $\frac{x}{2} + \frac{x + 4}{3} = 13$

Multiply every term by 6:

$$3x + 2(x + 4) = 78$$

$$3x + 2x + 8 = 78$$

$$5x + 8 = 78$$

$$5x = 70$$

$$x = 14$$

Check: $14 \div 2 = 7$ and $18 \div 3 = 6$, and $7 + 6 = 13$ ✓

Answer: 14 years

Marks: 1 mark for forming the equation. 1 mark for multiplying by 6. 1 mark for $5x = 70$. 1 mark for the correct answer.
