

Linear and Quadratic Simultaneous Equations

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Solve the simultaneous equations

$$y = x^2$$

$$x + y = 6$$

Substitute $y = x^2$ into $x + y = 6$:

$$x + x^2 = 6$$

Rearrange so one side is 0: $x^2 + x - 6 = 0$

Factorise: $(x + 3)(x - 2) = 0$

So $x = 2$ or $x = -3$

Find each y using $y = x^2$:

When $x = 2$: $y = 2^2 = 4$

When $x = -3$: $y = (-3)^2 = 9$

Answer: $x = 2, y = 4$ and $x = -3, y = 9$

Tip: You are not finished when you have the x values. Find the matching y value for each one and write them as pairs.

Marks: 1 mark for substituting to get a quadratic in one letter. 1 mark for $(x + 3)(x - 2)$. 1 mark for both x values. 1 mark for both y values.

2 (4 marks)

A curve has the equation $y = x^2 - 3x + 5$

A straight line has the equation $y = 2x + 1$

Work out the two points of intersection of the curve and the straight line.

At the points of intersection, both equations give the same y .

$$\text{So } x^2 - 3x + 5 = 2x + 1$$

$$\text{Subtract } 2x \text{ and } 1 \text{ from both sides: } x^2 - 5x + 4 = 0$$

$$\text{Factorise: } (x - 1)(x - 4) = 0$$

$$\text{So } x = 1 \text{ or } x = 4$$

Find each y using the line $y = 2x + 1$:

$$\text{When } x = 1: y = 2 \times 1 + 1 = 3$$

$$\text{When } x = 4: y = 2 \times 4 + 1 = 9$$

Answer: (1, 3) and (4, 9)

Marks: 1 mark for putting the two expressions for y equal. 1 mark for $(x - 1)(x - 4)$. 1 mark for both x values. 1 mark for both points.

3 (5 marks)

Solve the simultaneous equations

$$x^2 + y^2 = 25$$

$$y = x + 1$$

Substitute $y = x + 1$ into $x^2 + y^2 = 25$:

$$x^2 + (x + 1)^2 = 25$$

$$\text{Expand: } (x + 1)^2 = x^2 + 2x + 1$$

$$x^2 + x^2 + 2x + 1 = 25$$

$$2x^2 + 2x - 24 = 0$$

$$\text{Divide every term by 2: } x^2 + x - 12 = 0$$

$$\text{Factorise: } (x + 4)(x - 3) = 0$$

$$\text{So } x = 3 \text{ or } x = -4$$

Find each y using $y = x + 1$:

When $x = 3$: $y = 4$

When $x = -4$: $y = -3$

Answer: $x = 3, y = 4$ and $x = -4, y = -3$

Tip: $(x + 1)^2$ is $x^2 + 2x + 1$, not $x^2 + 1$. Write out $(x + 1)(x + 1)$ if you are not sure.

Marks: 1 mark for substituting $y = x + 1$. 1 mark for expanding $(x + 1)^2$ correctly. 1 mark for $(x + 4)(x - 3)$. 1 mark for both x values. 1 mark for both y values.

4 (3 marks)

$(3, -1)$ is one solution of the simultaneous equations

$$x^2 + y^2 = k$$

$$y = mx + 5$$

Work out the value of k and the value of m .

A solution of simultaneous equations fits **both** equations.

So put $x = 3$ and $y = -1$ into each equation.

First equation: $3^2 + (-1)^2 = k$

$9 + 1 = k$, so $k = 10$

Second equation: $-1 = 3m + 5$

Subtract 5 from both sides: $-6 = 3m$

Divide by 3: $m = -2$

Answer: $k = 10, m = -2$

Tip: Take care with the negative: $(-1)^2 = +1$, so $k = 9 + 1 = 10$, not 8.

Marks: 1 mark for $k = 10$. 1 mark for $-1 = 3m + 5$. 1 mark for $m = -2$.

5 (4 marks)

A curve has equation $y = x^2 + 4x + 1$

A line has equation $y = 2x - 5$

Use algebra to show that the curve and the line do not meet.

If they meet, both equations give the same y :

$$x^2 + 4x + 1 = 2x - 5$$

Subtract $2x$ from both sides and add 5 to both sides:

$$x^2 + 2x + 6 = 0$$

Complete the square: $(x + 1)^2 - 1 + 6 = 0$

$$(x + 1)^2 + 5 = 0, \text{ so } (x + 1)^2 = -5$$

A square can never be negative, so this equation has no solutions.

There is no value of x where the curve and the line meet, so they do not meet.

(Or use $b^2 - 4ac = 2^2 - 4 \times 1 \times 6 = -20$, which is negative, so there are no solutions.)

Answer: $(x + 1)^2 = -5$ has no solutions, so the curve and the line do not meet

Tip: Put the two equations together **first**, then show the quadratic has no solutions. Finish with a sentence that answers the question: "so the curve and the line do not meet".

Marks: 1 mark for putting the two expressions equal. 1 mark for $x^2 + 2x + 6 = 0$. 1 mark for $(x + 1)^2 + 5$ (or $b^2 - 4ac = -20$). 1 mark for explaining that there are no solutions, so they do not meet.

6 (8 marks)

The line $y = x + 4$ meets the circle $x^2 + y^2 = 58$ at the points P and Q .

(a) Show that the x -coordinates of P and Q satisfy the equation

$$x^2 + 4x - 21 = 0$$

Substitute $y = x + 4$ into $x^2 + y^2 = 58$:

$$x^2 + (x + 4)^2 = 58$$

Expand: $(x + 4)^2 = x^2 + 8x + 16$

$$x^2 + x^2 + 8x + 16 = 58$$

$$2x^2 + 8x - 42 = 0$$

Divide every term by 2:

$$x^2 + 4x - 21 = 0 \text{ as required.}$$

Answer: Shown

Marks: 1 mark for substituting $y = x + 4$. 1 mark for $(x + 4)^2 = x^2 + 8x + 16$. 1 mark for fully correct working.

- (b) Work out the length of PQ .
Give your answer in the form $a\sqrt{2}$ where a is an integer.
You must show your working.

Factorise the equation from part (a): $(x + 7)(x - 3) = 0$

So $x = 3$ or $x = -7$

Find each y using the line $y = x + 4$:

$x = 3$ gives $y = 7$, and $x = -7$ gives $y = -3$

So the points are $(3, 7)$ and $(-7, -3)$.

Across: $3 - (-7) = 10$. Up: $7 - (-3) = 10$

Use Pythagoras: $PQ^2 = 10^2 + 10^2 = 200$

$$PQ = \sqrt{200} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$$

Answer: $10\sqrt{2}$

Tip: Find y from the **line**, not the circle. The circle gives two y values for each x (for $x = 3$ it gives $y = 7$ or $y = -7$), but only one of them is on the line.

Marks: 1 mark for $(x + 7)(x - 3)$. 1 mark for both x values. 1 mark for both y values. 1 mark for $10^2 + 10^2$. 1 mark for $10\sqrt{2}$.