

# Iteration

For AQA GCSE Mathematics (8300) • Higher tier

Name \_\_\_\_\_

Total marks: **24**

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Answer **all** questions in the spaces provided. Show your working: marks are given for correct method. You may use a calculator.

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- 1** A sequence of numbers is formed by the iterative process

$$u_{n+1} = 2u_n - 3 \quad u_1 = 5$$

Work out the values of  $u_2$  and  $u_3$

**[2 marks]**

Answer  $u_2 =$  \_\_\_\_\_  $u_3 =$  \_\_\_\_\_

- 2** Use the iteration formula  $x_{n+1} = 3 - \frac{1}{x_n}$  with  $x_1 = 2$  to work out  $x_2$ ,  $x_3$  and  $x_4$   
Give  $x_4$  to 4 decimal places.

**[3 marks]**

Answer  $x_4 =$  \_\_\_\_\_

- 3 Here is a formula for an iterative process.

$$u_{n+1} = \frac{30}{u_n} + 2$$

$$u_2 = 8$$

Work out the values of  $u_1$  and  $u_3$

[3 marks]

Answer  $u_1 = \underline{\hspace{2cm}}$   $u_3 = \underline{\hspace{2cm}}$

- 4 Which equation can be rearranged to give  $x = \sqrt{5x - 2}$ ?

Circle your answer.

[1 mark]

$$x^2 - 5x + 2 = 0$$

$$x^2 + 5x - 2 = 0$$

$$x^2 - 5x - 2 = 0$$

$$x^2 = 5x + 2$$

- 5 (a) Show that the equation  $x^3 - 6x + 2 = 0$  has a solution between  $x = 0$  and  $x = 1$   
[2 marks]

- (b) Show that  $x^3 - 6x + 2 = 0$  can be rearranged to give  $x = \frac{x^3 + 2}{6}$   
[1 mark]

- (c) Starting with  $x_1 = 0$ , use the iteration formula  $x_{n+1} = \frac{x_n^3 + 2}{6}$  three times to find an estimate for the solution.  
Give your answer to 4 decimal places.  
[3 marks]

Answer \_\_\_\_\_

- 6 Sanjay has £3000 in a savings account.  
At the end of each year, the bank adds interest and then Sanjay takes out £400.  
The amount in the account at the end of  $n$  years is  $S_n$  pounds, where

$$S_{n+1} = 1.04 \times S_n - 400 \quad S_0 = 3000$$

At the end of which year will the amount in the account first be less than £2000?  
You must show your working.

[3 marks]

Answer year \_\_\_\_\_

- 7** A rectangle has width  $w$  cm and length  $(w + 3)$  cm.  
 The area of the rectangle is  $50 \text{ cm}^2$   
 An approximate value of  $w$  can be found using the iterative formula

$$w_{n+1} = \sqrt{50 - 3w_n}$$

The starting value is  $w_1 = 5$

- (a) Work out the values of  $w_2$  and  $w_3$

**[2 marks]**

Answer  $w_2 = \underline{\hspace{2cm}}$   $w_3 = \underline{\hspace{2cm}}$

- (b) Keep using the iterative formula until two values in a row round to the same number to 1 decimal place.

Write down the width of the rectangle to 1 decimal place.

**[1 mark]**

Answer  $\underline{\hspace{2cm}}$  cm

**8**

$$x_{n+1} = 6 - \frac{2}{x_n}$$

Use  $x_1 = 1$  to work out an approximate solution to  $x = 6 - \frac{2}{x}$   
 Give your answer to 4 significant figures.

**[3 marks]**

Answer  $x = \underline{\hspace{2cm}}$