

Inequalities and Regions on Graphs

Solutions

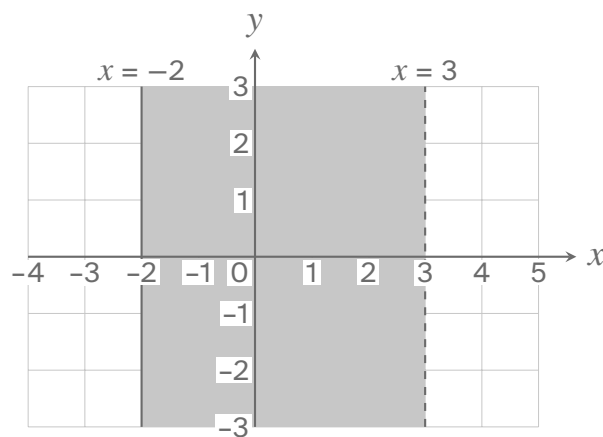
For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The grid shows the lines $x = -2$ and $x = 3$

The region between the lines is shaded.



Which inequality describes the shaded region?

Tick **one** box.

The shaded region is between the vertical lines $x = -2$ and $x = 3$, so the inequality is about x .

The line $x = -2$ is solid, so $x = -2$ is included: $-2 \leq x$

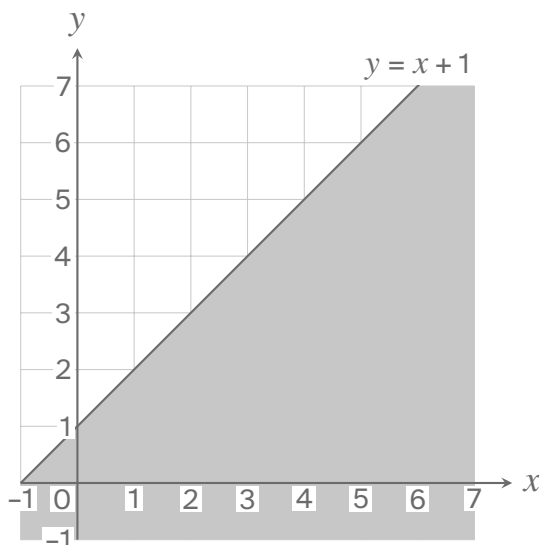
The line $x = 3$ is dashed, so $x = 3$ is not included: $x < 3$

Answer: $-2 \leq x < 3$

Tip: Vertical lines have equations $x = \dots$, so a region between two vertical lines is described using x , not y .

2 (2 marks)

The line $y = x + 1$ is drawn on the grid. The region below the line is shaded.



- (a) Write down the inequality that describes the shaded region.

The shaded region is below the line $y = x + 1$.

The line is solid, so points on the line are included.

So use $\leq$.

Answer: $y \leq x + 1$

- (b) Does the point $(2, 3)$ lie in the shaded region?

Tick a box.

Give a reason for your answer.

Substitute $x = 2$ into $x + 1$: $2 + 1 = 3$

So $(2, 3)$ lies **on** the line $y = x + 1$.

The line is solid, so points on it are in the region.

Answer: Yes: $3 \leq 2 + 1$ (the point is on the solid boundary line)

Tip: A point on a solid boundary line **is** in the region. A point on a dashed line is not.

3 (3 marks)

The lines $y = 1$, $x + y = 6$ and $y = 2x$ are drawn on the grid.

On the grid, shade the region that satisfies all of these inequalities.

Label the region R .

$$y \geq 1 \quad x + y \leq 6 \quad y \leq 2x$$

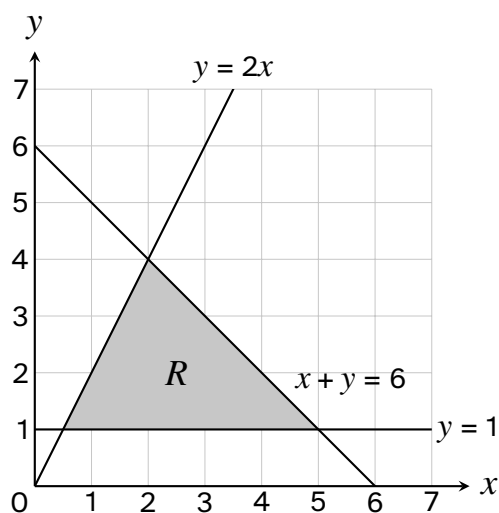
Use each line to decide which side you need:

$y \geq 1$: on or above the line $y = 1$.

$x + y \leq 6$: on or below the line $x + y = 6$.

$y \leq 2x$: on or below the line $y = 2x$.

R is the triangle where all three overlap, with vertices $(0.5, 1)$, $(5, 1)$ and $(2, 4)$.



Check with a point inside, e.g. $(2, 2)$:

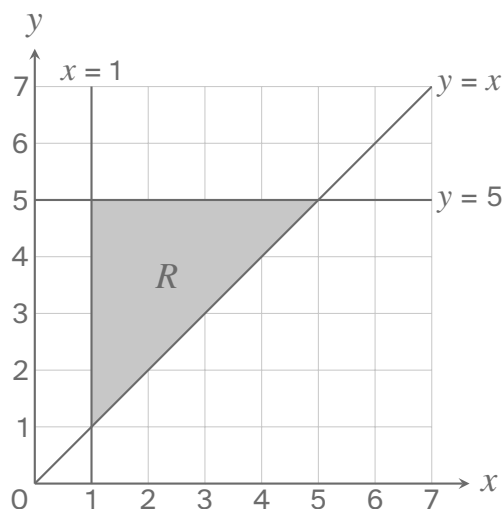
$$2 \geq 1 \quad \checkmark \quad 2 + 2 = 4 \leq 6 \quad \checkmark \quad 2 \leq 2 \times 2 \quad \checkmark$$

Answer: Triangle with vertices $(0.5, 1)$, $(5, 1)$, $(2, 4)$

Marks: 1 mark for each boundary used correctly. 3 marks for the correct region.

4 (3 marks)

The region R is shown on the grid.



Write down the three inequalities that define the shaded region R .

R is to the right of $x = 1$: $x \geq 1$

R is above the line $y = x$: $y \geq x$

R is below the line $y = 5$: $y \leq 5$

All three lines are solid, so use $\geq$ and $\leq$.

Answer: $x \geq 1, y \geq x, y \leq 5$

Marks: 1 mark for each correct inequality.

5 (3 marks)

On the grid, mark with a cross (×) each point with integer coordinates that satisfies all three inequalities.

$$x > 0 \quad y > 0 \quad x + 2y < 6$$

$x > 0$ and $y > 0$: the points must be to the right of the y -axis and above the x -axis, not on the axes.

Draw $x + 2y = 6$ as a dashed line. It goes through $(0, 3)$ and $(6, 0)$.

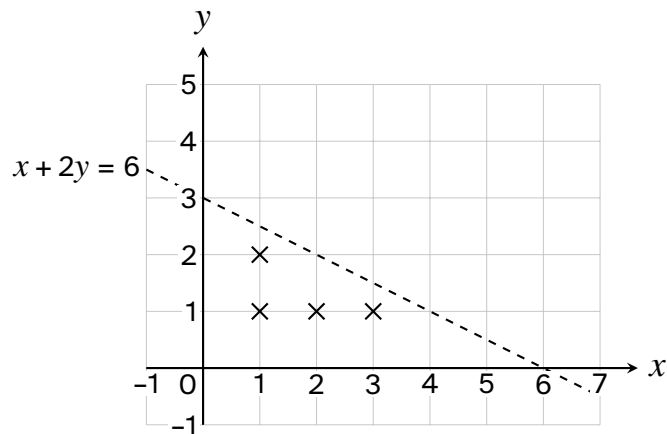
The points must be below this line, not on it.

Check each row:

$y = 1$: $x + 2 < 6$, so $x < 4$. Points $(1, 1)$, $(2, 1)$, $(3, 1)$

$y = 2$: $x + 4 < 6$, so $x < 2$. Point $(1, 2)$

$y = 3$: $x + 6 < 6$, so $x < 0$. No points



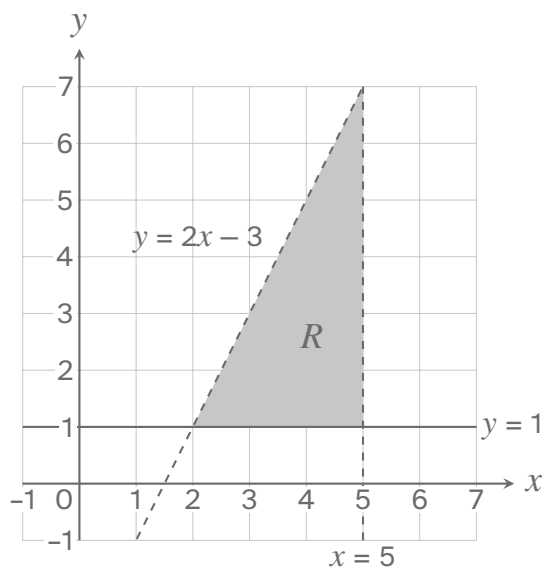
Answer: $(1, 1)$, $(2, 1)$, $(3, 1)$, $(1, 2)$

Tip: All three inequalities are strict, so points on the axes or on the line do not count. For example, $(2, 2)$ gives $2 + 4 = 6$, which is not less than 6.

Marks: 3 marks for all four points and no extras. 2 marks for three correct points. 1 mark for drawing the line $x + 2y = 6$.

6 (2 marks)

The region R is shaded on the grid.



Tomasz says that R is represented by

$$y > 1 \quad \text{and} \quad x < 5 \quad \text{and} \quad y > 2x - 3$$

Write down **two** mistakes that Tomasz has made.

The line $y = 1$ is solid, so points on it are included.

Tomasz should have written $y \geq 1$, not $y > 1$.

R is below (to the right of) the line $y = 2x - 3$.

Tomasz should have written $y < 2x - 3$, not $y > 2x - 3$.

($x < 5$ is correct: R is to the left of the dashed line $x = 5$.)

Answer: $y > 1$ should be $y \geq 1$; $y > 2x - 3$ should be $y < 2x - 3$

Tip: Check each inequality with a point inside R , such as $(4, 2)$: $2 > 2 \times 4 - 3 = 5$ is false, so $y > 2x - 3$ must be wrong.

Marks: 1 mark for each correct mistake.

7 (4 marks)

On the grid, identify the region represented by

$$x \leq 4 \quad \text{and} \quad y > -x \quad \text{and} \quad y \leq \frac{1}{2}x + 3$$

Label the region R .

Use dashed lines for $<$ or $>$, and solid lines for $\leq$ or $\geq$.

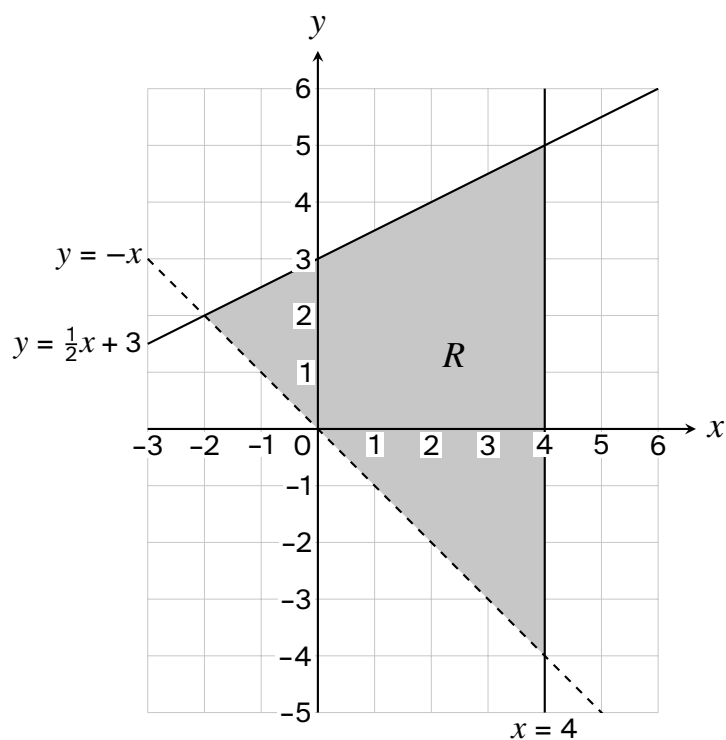
$x = 4$ is a vertical line. Draw it solid.

$y = -x$ goes through $(0, 0)$ with gradient -1 . Draw it dashed.

$y = \frac{1}{2}x + 3$ goes through $(0, 3)$ and $(4, 5)$. Draw it solid.

R is on or left of $x = 4$, above $y = -x$, and on or below $y = \frac{1}{2}x + 3$.

R is the triangle with vertices $(-2, 2)$, $(4, -4)$ and $(4, 5)$.



Check with (2, 1): $2 \leq 4$ ✓ $1 > -2$ ✓ $1 \leq 1 + 3$ ✓

Answer: Triangle with vertices $(-2, 2)$, $(4, -4)$, $(4, 5)$

Tip: To draw $y = \frac{1}{2}x + 3$, start at $(0, 3)$ and go 2 across and 1 up. Plotting a few points, such as $(-2, 2)$, $(0, 3)$ and $(4, 5)$, is safer than guessing.

Marks: 1 mark for each correct line ($x = 4$ solid, $y = -x$ dashed, $y = \frac{1}{2}x + 3$ solid). 1 mark for the correct region labelled R .

8 (3 marks)

On the grid, identify the region represented by

$$-1 \leq x < 3 \quad \text{and} \quad 2 < y \leq 7 - x$$

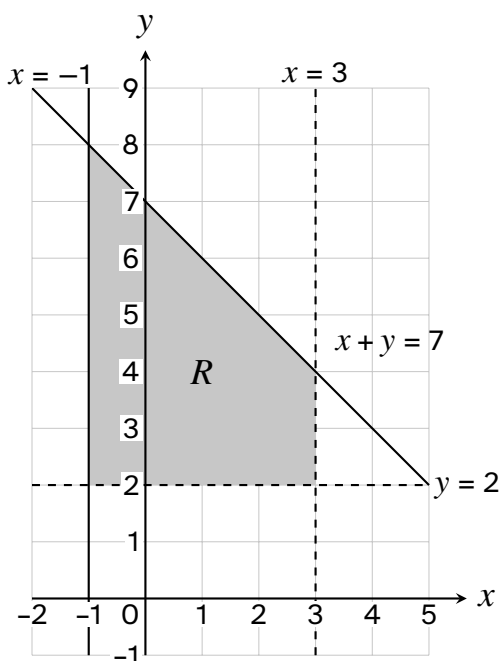
Label the region R .

$-1 \leq x < 3$: draw $x = -1$ solid and $x = 3$ dashed. R is between them.

$2 < y$: draw $y = 2$ dashed. R is above it.

$y \leq 7 - x$: draw $x + y = 7$ solid, through $(0, 7)$ and $(5, 2)$. R is on or below it.

R is the four-sided shape with vertices $(-1, 2)$, $(3, 2)$, $(3, 4)$ and $(-1, 8)$.



Check with (1,4): $-1 \leq 1 < 3$ ✓ $2 < 4 \leq 6$ ✓

Answer: Region with vertices $(-1, 2)$, $(3, 2)$, $(3, 4)$, $(-1, 8)$

Tip: $-1 \leq x < 3$ is two inequalities in one: $x \geq -1$ **and** $x < 3$. The same goes for $2 < y \leq 7 - x$.

Marks: 1 mark for $x = -1$ and $x = 3$ correct. 1 mark for $y = 2$ and $x + y = 7$ correct. 1 mark for the correct region labelled R .
