

Graph Transformations

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The graph of $y = f(x)$ is translated to the graph of $y = f(x - 6)$
(4, -3) is a point on the translated graph.

Circle the point on the graph of $y = f(x)$ that moves to (4, -3)

Options: (10, -3), (-2, -3), (4, -9), (4, 3)

$f(x - 6)$ moves every point of the graph 6 to the **right**.

So the point that moves to (4, -3) started 6 to the left of it.

$$(4 - 6, -3) = (-2, -3)$$

Check: (-2, -3) moved 6 right is (4, -3) ✓

Answer: (-2, -3)

Tip: $(x - 6)$ inside the bracket moves the graph 6 to the right, so you go 6 to the **left** to get back to the original point. (10, -3) is a common wrong answer.

2 (2 marks)

Describe fully the single transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 3)^2$

$$y = (x - 3)^2 \text{ is } f(x - 3) \text{ where } f(x) = x^2.$$

$f(x - 3)$ moves the graph 3 to the **right**.

Check: the minimum point moves from (0, 0) to (3, 0) ✓

A movement like this is a translation, described by a column vector.

Answer: Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Tip: Use the word “translation” (not “move” or “transformation”) and give a column vector. A change inside the bracket goes the opposite way to its sign.

Marks: 1 mark for the word translation. 1 mark for the correct vector.

3 (4 marks)

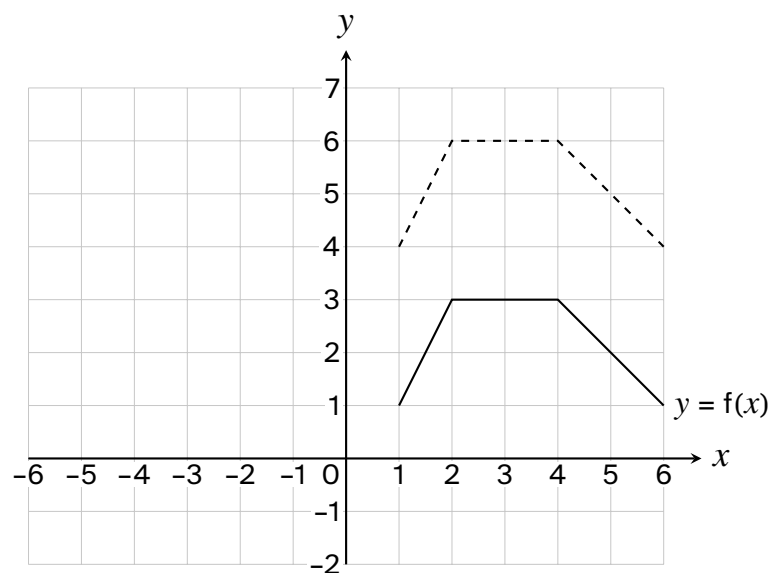
The graph of $y = f(x)$ is drawn on each grid.

(a) On the grid, draw the graph of $y = f(x) + 3$

$f(x) + 3$ adds 3 to every y -value, so the graph moves up 3.

The corners of the graph move to (1, 4), (2, 6), (4, 6), (6, 4)

Join them up (shown dashed):



Answer: The graph moved up 3

Marks: 2 marks for a correct graph. 1 mark for a translation up or down by a different amount.

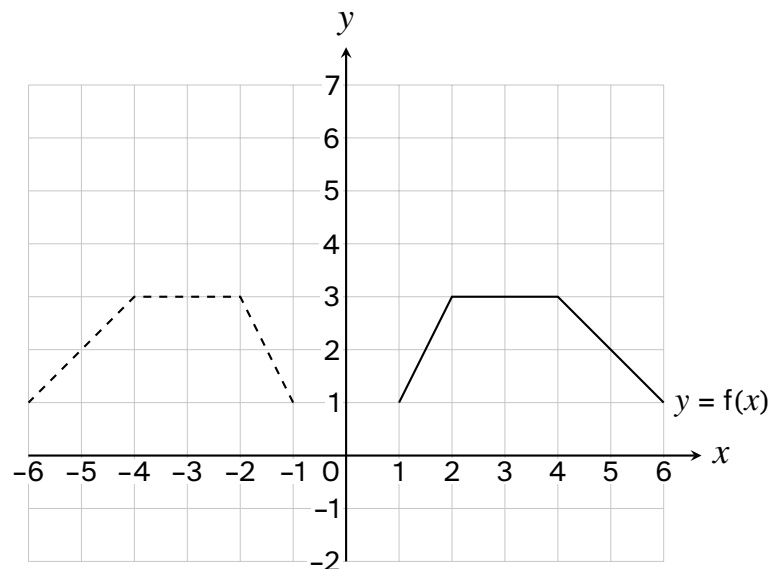
(b) On the grid, draw the graph of $y = f(-x)$

$f(-x)$ is a reflection in the y -axis.

Each point (x, y) moves to $(-x, y)$: the x -coordinate changes sign.

The corners move to $(-1, 1)$, $(-2, 3)$, $(-4, 3)$, $(-6, 1)$

Join them up (shown dashed):



Answer: The graph reflected in the y -axis

Tip: $f(-x)$ reflects in the y -axis. $-f(x)$ reflects in the x -axis. Do not rotate the graph.

Marks: 2 marks for a correct graph. 1 mark for three correct points.

4 (3 marks)

The curve $y = f(x)$ has a maximum point at $(-3, 4)$

Write down the coordinates of the maximum point of the curve

(a) $y = f(x) + 2$

$f(x) + 2$ moves the graph up 2.

$(-3, 4) \rightarrow (-3, 4 + 2) = (-3, 6)$

Answer: $(-3, 6)$

(b) $y = f(x - 5)$

$f(x - 5)$ moves the graph 5 to the **right**.

$$(-3, 4) \rightarrow (-3 + 5, 4) = (2, 4)$$

Answer: (2, 4)

Tip: A number inside the bracket moves the graph the opposite way to its sign: $(x - 5)$ moves it right.

(c) $y = f(-x)$

$f(-x)$ is a reflection in the y -axis: the x -coordinate changes sign.

$$(-3, 4) \rightarrow (3, 4)$$

Answer: (3, 4)

5 (2 marks)

- (a) The graph of $y = \frac{1}{x}$ is translated by the vector $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
Write down the equation of the translated graph.

The vector $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ moves the graph 5 to the right and not up or down.

5 to the right means $f(x - 5)$: replace x with $(x - 5)$.

$$y = \frac{1}{x - 5}$$

Answer: $y = \frac{1}{x - 5}$

Tip: Moving right uses $(x - 5)$, not $(x + 5)$. $y = \frac{1}{x} - 5$ would move the graph down, not right.

- (b) The graph of $y = 2x + 7$ is reflected in the x -axis.
Write down the equation of the reflected graph.

Reflecting in the x -axis means $y = -f(x)$: every y -value changes sign.

$$y = -(2x + 7)$$

$$y = -2x - 7$$

Check: $(0, 7)$ on the first line reflects to $(0, -7)$ on the second ✓

Answer: $y = -2x - 7$

Tip: Change the sign of the **whole** right-hand side. Replacing x with $-x$ would be a reflection in the y -axis.

6 (2 marks)

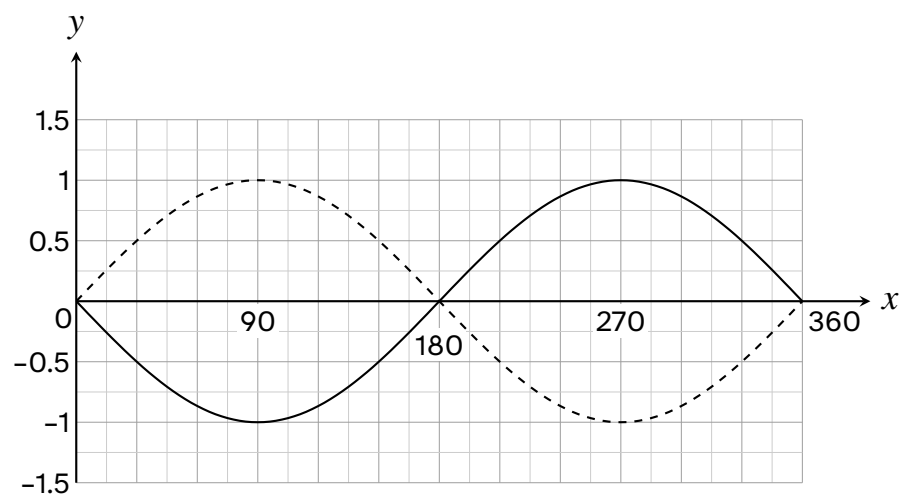
The graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ is shown as a dashed line on the grid.

(a) On the grid below, draw the graph of $y = -\sin x$ for $0^\circ \leq x \leq 360^\circ$

$-\sin x$ is $-f(x)$, a reflection in the x -axis: every y -value changes sign.

$(90, 1) \rightarrow (90, -1)$, $(270, -1) \rightarrow (270, 1)$

$(0, 0)$, $(180, 0)$ and $(360, 0)$ stay where they are.



Answer: The curve through $(0, 0)$, $(90, -1)$, $(180, 0)$, $(270, 1)$, $(360, 0)$

Tip: $-\sin x$ reflects the graph in the x -axis (upside down). It does not move it sideways.

- (b) The graph of $y = \sin x$ is translated by the vector $\begin{pmatrix} 90 \\ 2 \end{pmatrix}$

Write down the coordinates of the image of the maximum point of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$

The maximum point of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ is $(90, 1)$.

The vector moves every point 90 to the right and 2 up.

$$(90, 1) \rightarrow (90 + 90, 1 + 2) = (180, 3)$$

Answer: $(180, 3)$

Tip: The top number of the vector changes the x -coordinate; the bottom number changes the y -coordinate.

7 (3 marks)

The graph of $y = x^2$ is translated to give graph B .

Graph B has equation $y = x^2 - 8x + 19$

Work out the vector of the translation.

Complete the square: $x^2 - 8x = (x - 4)^2 - 16$

$$\text{So } x^2 - 8x + 19 = (x - 4)^2 - 16 + 19 = (x - 4)^2 + 3$$

$(x - 4)^2$ moves the graph 4 to the right.

$+3$ moves the graph 3 up.

Check: the minimum point moves from $(0, 0)$ to $(4, 3)$ ✓

Answer: $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Tip: $(x - 4)^2$ moves the graph 4 to the **right**, so the top number of the vector is $+4$, not -4 .

Marks: 1 mark for $(x - 4)^2$. 1 mark for $(x - 4)^2 + 3$ or the minimum point $(4, 3)$. 1 mark for the correct vector.

8 (4 marks)

The curve $y = x^3 - 4x$ is translated 1 unit to the left and 3 units up.

The translated curve has equation $y = g(x)$

Show that $g(x) = x^3 + 3x^2 - x$

1 unit to the left means replacing x with $(x + 1)$.

3 units up means adding 3.

$$g(x) = (x + 1)^3 - 4(x + 1) + 3$$

First expand two brackets: $(x + 1)^2 = x^2 + 2x + 1$

Now multiply by $(x + 1)$ again:

$$(x^2 + 2x + 1)(x + 1) = x^3 + x^2 + 2x^2 + 2x + x + 1 = x^3 + 3x^2 + 3x + 1$$

Expand the other bracket: $-4(x + 1) = -4x - 4$

$$g(x) = x^3 + 3x^2 + 3x + 1 - 4x - 4 + 3$$

$$= x^3 + 3x^2 - x, \text{ as required.}$$

Tip: Replace **every** x with $(x + 1)$, including the x in $-4x$. $(x + 1)^3$ is not $x^3 + 1$.

Marks: 1 mark for $(x + 1)^3 - 4(x + 1)$. 1 mark for adding 3 (a complete expression for $g(x)$). 1 mark for expanding $(x + 1)^3$ correctly. 1 mark for a fully correct method leading to $x^3 + 3x^2 - x$.
