

Gradients and Areas Under Graphs

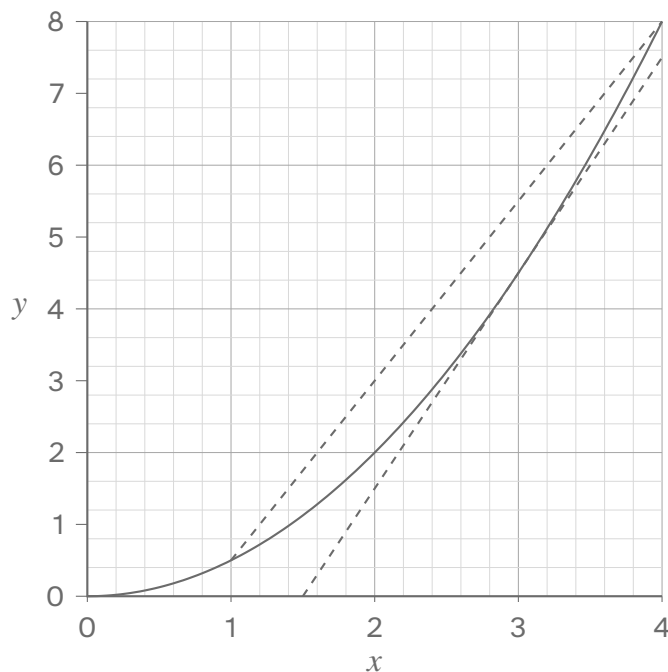
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (8 marks)

Here is the graph of $y = \frac{1}{2}x^2$ for $0 \leq x \leq 4$ (chord and tangent dashed)



- (a) Work out the gradient of the chord joining the points on the curve where $x = 1$ and $x = 4$

A chord is a straight line joining two points on the curve.

$$\text{When } x = 1, y = \frac{1}{2} \times 1^2 = 0.5$$

$$\text{When } x = 4, y = \frac{1}{2} \times 4^2 = 8$$

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{8 - 0.5}{4 - 1}$$

$$= \frac{7.5}{3} = 2.5$$

Answer: 2.5

Marks: 1 mark for change in $y \div$ change in x . 1 mark for 2.5.

- (b) By drawing a tangent, estimate the gradient of the curve at $x = 3$

Draw a tangent: a straight line that just touches the curve at (3, 4.5) (the dashed line).

Pick two points on the tangent that are far apart, e.g. (1.5, 0) and (4, 7.5).

$$\text{Gradient} = \frac{7.5 - 0}{4 - 1.5} = \frac{7.5}{2.5}$$

$$= 3$$

Answers close to 3 are fine, because the tangent is drawn by eye.

Answer: about 3 (accept 2.6 to 3.4)

Tip: You must draw a tangent. Working out $4.5 \div 3$ from the point on the curve is wrong and gets no marks.

Marks: 1 mark for a tangent drawn at $x = 3$ and its gradient worked out. 1 mark for an answer in the accepted range.

- (c) Use 4 strips of equal width to estimate the area under the curve between $x = 0$ and $x = 4$

4 strips from $x = 0$ to $x = 4$, so each strip is 1 wide.

Heights at $x = 0, 1, 2, 3, 4$ are 0, 0.5, 2, 4.5, 8

Each strip is a trapezium: area = $\frac{1}{2}(a + b) \times 1$

$$\frac{1}{2}(0 + 0.5) + \frac{1}{2}(0.5 + 2) + \frac{1}{2}(2 + 4.5) + \frac{1}{2}(4.5 + 8)$$

$$= 0.25 + 1.25 + 3.25 + 6.25$$

= 11

Answer: 11

Tip: Use exactly the number of strips asked for, and use trapezia (not rectangles). Read the heights carefully from the scale.

Marks: 1 mark for the correct heights. 1 mark for adding the trapezium areas. 1 mark for 11.

- (d) Is your answer to part (c) an overestimate or an underestimate?
Give a reason for your answer.

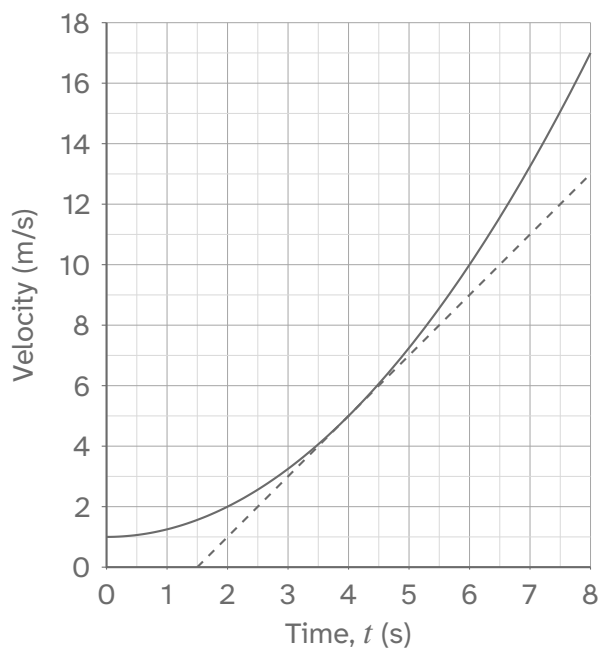
The curve bends below the straight tops of the trapezia.

So the trapezia include some extra area above the curve.

Answer: Overestimate: the trapezia are above the curve

2 (6 marks)

The graph shows the velocity, v m/s, of an object for the first 8 seconds (tangent at $t = 4$ dashed).



- (a) Use 4 strips of equal width to estimate the area under the graph between $t = 0$ and $t = 8$

4 strips from $t = 0$ to $t = 8$, so each strip is 2 wide.

Values at $t = 0, 2, 4, 6, 8$ are 1, 2, 5, 10, 17

Each strip is a trapezium: area = $\frac{1}{2}(a + b) \times 2 = a + b$

$$(1 + 2) + (2 + 5) + (5 + 10) + (10 + 17)$$

$$= 3 + 7 + 15 + 27$$

$$= 52$$

Answer: 52

Marks: 1 mark for the correct heights. 1 mark for adding the trapezium areas. 1 mark for 52.

- (b) What does the area under the graph represent?

Area = velocity $\times$ time, which is a distance.

So the area under a velocity-time graph is the distance travelled.

Answer: The distance travelled (in metres)

Tip: Area under a velocity-time graph = distance. Gradient of a velocity-time graph = acceleration. Do not mix them up.

- (c) Estimate the acceleration at $t = 4$
You must show your working.

Acceleration is the gradient of the velocity-time graph.

Draw a tangent at (4, 5) (the dashed line).

Two points on the tangent: (1.5, 0) and (8, 13).

$$\text{Gradient} = \frac{13 - 0}{8 - 1.5} = \frac{13}{6.5}$$

$$= 2$$

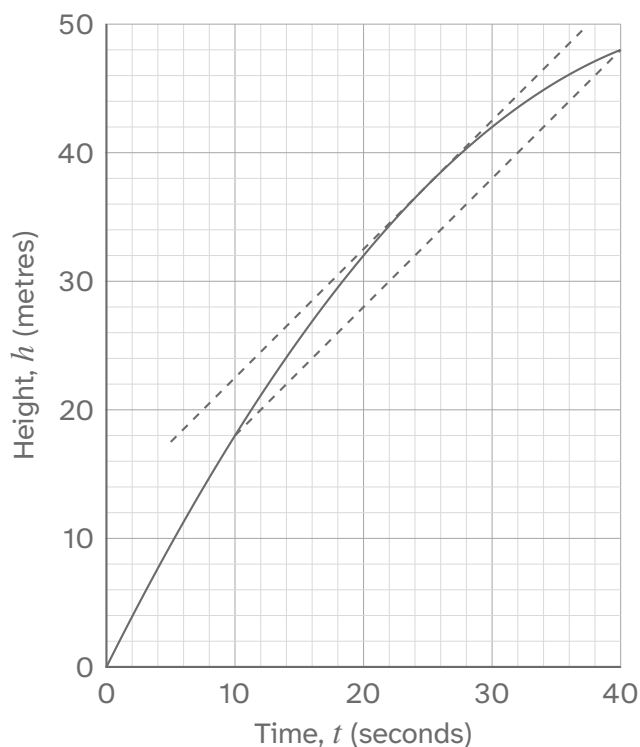
Answer: about 2 (accept 1.6 to 2.4)

Marks: 1 mark for a tangent at $t = 4$ and its gradient worked out. 1 mark for an answer in the accepted range.

3 (4 marks)

A drone takes off from the ground and rises vertically.

The graph shows its height, h metres, t seconds after take-off (chord and parallel tangent dashed).



- (a) Work out the average rate at which the height of the drone increases between $t = 10$ and $t = 40$

Give the units of your answer.

Average rate of change = change in height $\div$ change in time

At $t = 10$ the height is 18 m. At $t = 40$ the height is 48 m.

$$(48 - 18) \div (40 - 10) = 30 \div 30$$

$$= 1$$

The units are metres per second (m/s).

Answer: 1 m/s

Tip: This is the gradient of the chord from (10, 18) to (40, 48). Use the heights at $t = 10$ and $t = 40$, not the height at $t = 40$ divided by 40.

Marks: 1 mark for 1. 1 mark for the units m/s.

- (b) Estimate a value of t at which the instantaneous rate of increase of the height is the same as your answer to part (a).
You must show your working on the graph.

The instantaneous rate of increase is the gradient of a tangent.

Draw the chord from (10, 18) to (40, 48).

Then find where a tangent to the curve is parallel to the chord (both dashed on the graph).

This happens at about $t = 25$

Answer: about 25 seconds (accept 21 to 29)

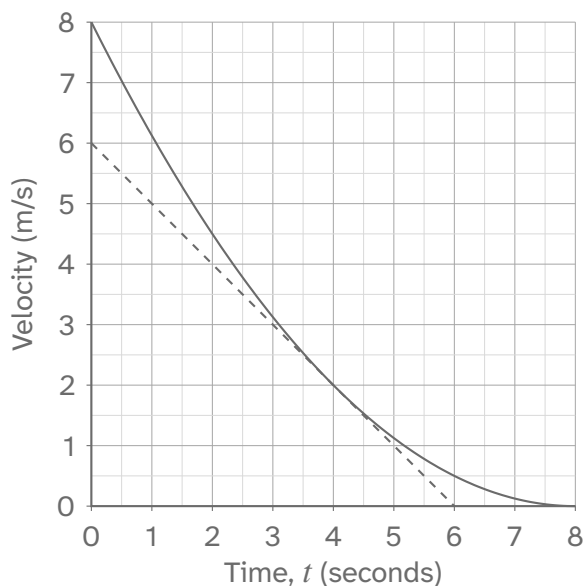
Tip: The tangent must be parallel to the chord between $t = 10$ and $t = 40$, not the chord from the origin.

Marks: 1 mark for a tangent drawn parallel to the chord. 1 mark for an answer in the accepted range.

4 (6 marks)

A sledge slides across flat snow and slows down.

The graph shows the velocity of the sledge t seconds after it reaches the flat snow (tangent at $t = 4$ dashed).



- (a) Work out an estimate for the deceleration of the sledge at $t = 4$
You must show your working.

Deceleration is found from the gradient of the velocity-time graph.

At $t = 4$ the velocity is 2 m/s.

Draw a tangent at (4, 2) (the dashed line).

Two points on the tangent: (0, 6) and (6, 0).

$$\text{Gradient} = \frac{0 - 6}{6 - 0} = -1$$

The gradient is negative because the sledge is slowing down.

So the deceleration is about 1 m/s^2 .

Answer: about 1 m/s^2 (accept 0.8 to 1.2)

Tip: You must draw a tangent. Dividing the velocity by the time, $2 \div 4$, is wrong. A deceleration of 1 m/s^2 is the same as an acceleration of -1 m/s^2 .

Marks: 1 mark for a tangent at $t = 4$ and its gradient worked out. 1 mark for an answer in the accepted range.

- (b) Use 4 strips of equal width to work out an estimate for the distance the sledge travels before it stops.

The distance is the area under the velocity-time graph.

The sledge stops at $t = 8$, so use 4 strips from 0 to 8, each 2 wide.

Heights at $t = 0, 2, 4, 6, 8$ are 8, 4.5, 2, 0.5, 0

Each strip is a trapezium: area = $\frac{1}{2}(a + b) \times 2 = a + b$

$$(8 + 4.5) + (4.5 + 2) + (2 + 0.5) + (0.5 + 0)$$

$$= 12.5 + 6.5 + 2.5 + 0.5 = 22$$

Answer: 22 m

Tip: Read the heights carefully: each small square up is 0.5 m/s. The last strip is a triangle.

Marks: 1 mark for the correct heights. 1 mark for adding the trapezium areas. 1 mark for 22.

- (c) Is your estimate from part (b) an overestimate or an underestimate?

Tick a box.

Give a reason for your answer.

The curve lies below the straight tops of the trapezia.

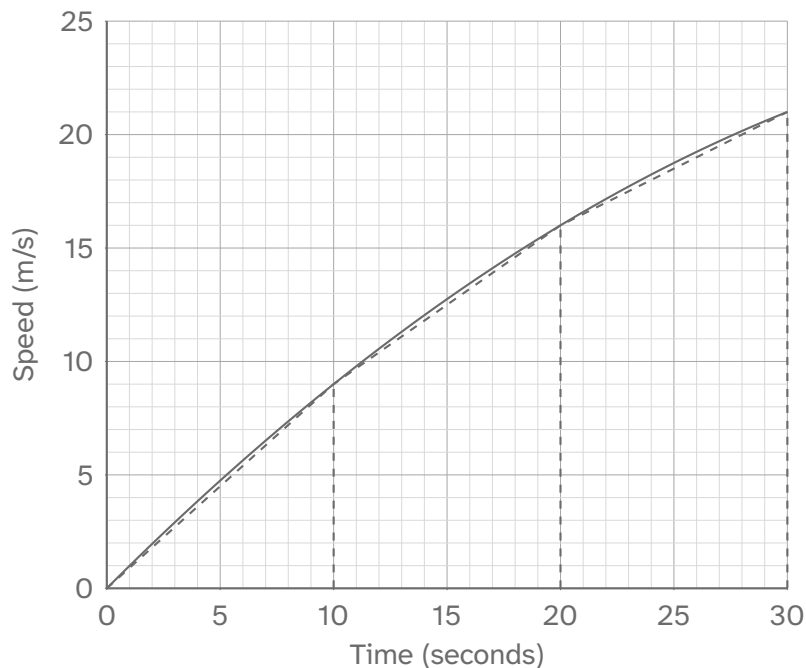
So the trapezia include some extra area.

Answer: Overestimate: the trapezia are above the curve

5 (4 marks)

A train leaves a station.

The speed-time graph shows the first 30 seconds of its journey (the 3 strips are dashed).



- (a) Use 3 strips of equal width to estimate the distance the train travels in these 30 seconds.

The distance is the area under the speed-time graph.

3 strips from 0 to 30 seconds, so each strip is 10 seconds wide.

Speeds at 0, 10, 20, 30 seconds are 0, 9, 16, 21 m/s

Strip 1 (a triangle): $\frac{1}{2} \times 10 \times 9 = 45$

Strip 2: $\frac{1}{2}(9 + 16) \times 10 = 125$

Strip 3: $\frac{1}{2}(16 + 21) \times 10 = 185$

Total: $45 + 125 + 185 = 355$

Answer: 355 m

Tip: Distance is the **area** under the graph. Do not just multiply the final speed by the time: 21×30 is far too big.

Marks: 1 mark for the correct speeds. 1 mark for adding the areas of the strips. 1 mark for 355.

- (b) Mira says,
 “If you used 6 strips of equal width instead of 3, the estimate
 would be larger.”
Is Mira correct?
Tick **one** box.
Give a reason for your answer.

The curve bends above the straight tops of the strips, so 3 strips miss some area.

With 6 strips the tops are closer to the curve, so less area is missed.

The estimate gets bigger (closer to the real distance).

Answer: Yes: the strips are below the curve, and more strips miss less of the area

Tip: Here the straight tops of the strips are always below the curve, so every estimate is too small. More strips give a bigger, better estimate.