

Function Notation

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

$$f(x) = 5 - 2x^2$$

(a) Work out $f(3)$

$f(3)$ means replace x with 3.

$$f(3) = 5 - 2 \times 3^2$$

$$= 5 - 2 \times 9$$

$$= 5 - 18 = -13$$

Answer: -13

Tip: Square first, then multiply by 2, then subtract from 5. Working out $(5 - 2) \times 9$ is a common mistake.

(b) Work out $f(-\frac{1}{2})$

$$f(-\frac{1}{2}) = 5 - 2 \times (-\frac{1}{2})^2$$

$$(-\frac{1}{2})^2 = \frac{1}{4}$$

$$= 5 - 2 \times \frac{1}{4} = 5 - \frac{1}{2} = 4\frac{1}{2}$$

Answer: $4\frac{1}{2}$ (or 4.5)

(c) Solve $f(x) = -27$

$$f(x) = -27 \text{ means } 5 - 2x^2 = -27.$$

$$\text{Subtract 5 from both sides: } -2x^2 = -32$$

Divide both sides by -2 : $x^2 = 16$

Square root: $x = 4$ or $x = -4$

Answer: $x = 4$ or $x = -4$

Tip: A square root has two answers. Give the negative one as well.

Marks: 1 mark for $x^2 = 16$. 1 mark for both answers.

2 (1 mark)

$$f(x) = x^3 - x^2$$

Circle the value of $f(-2)$

Options: -12 , -4 , 4 , 12

$$f(-2) = (-2)^3 - (-2)^2$$

$$(-2)^3 = -2 \times -2 \times -2 = -8$$

$$(-2)^2 = -2 \times -2 = 4$$

$$-8 - 4 = -12$$

Answer: -12

Tip: A negative number cubed is negative, but a negative number squared is positive. Use brackets when you substitute.

3 (3 marks)

$$g(x) = \frac{x+5}{3}$$

(a) Work out $g(7)$

$$g(7) = \frac{7+5}{3}$$

$$= \frac{12}{3} = 4$$

Answer: 4

(b) Solve $g(x) = x - 1$

$$\frac{x+5}{3} = x - 1$$

Multiply both sides by 3: $x + 5 = 3(x - 1)$

$$x + 5 = 3x - 3$$

$$8 = 2x$$

$$x = 4$$

Answer: $x = 4$

Marks: 1 mark for $x + 5 = 3(x - 1)$. 1 mark for $x = 4$.

4 (2 marks)

$$h(x) = x^2 + 3x$$

Show that $h(x+1) = x^2 + 5x + 4$

Replace every x in $h(x)$ with $(x+1)$:

$$h(x+1) = (x+1)^2 + 3(x+1)$$

$$(x+1)^2 = x^2 + 2x + 1$$

$$3(x+1) = 3x + 3$$

$$\text{Add: } x^2 + 2x + 1 + 3x + 3 = x^2 + 5x + 4$$

Answer: Shown

Tip: Put $(x+1)$ in brackets every time you substitute it. Writing $3x + 1$ instead of $3(x+1)$ is a common mistake.

Marks: 1 mark for $(x+1)^2 + 3(x+1)$. 1 mark for expanding it correctly.

5 (3 marks)

$$f(x) = 3x - 2$$

Solve $f(2x) = f(x) + 10$

$f(2x)$ means replace x with $2x$:

$$f(2x) = 3(2x) - 2 = 6x - 2$$

Form the equation: $6x - 2 = (3x - 2) + 10$

$$6x - 2 = 3x + 8$$

$$3x = 10$$

$$x = \frac{10}{3}$$

Answer: $x = \frac{10}{3}$

Tip: $f(2x)$ is not the same as $2f(x)$. Here $f(2x) = 6x - 2$, but $2f(x) = 6x - 4$.

Marks: 1 mark for $f(2x) = 6x - 2$. 1 mark for forming the equation. 1 mark for $x = \frac{10}{3}$.

6 (3 marks)

$f(x) = ax + b$ where a and b are constants.

$$f(4) = 3 \times f(1) \quad \text{and} \quad f(2) = 10$$

Work out the values of a and b .

$$f(4) = 4a + b \quad \text{and} \quad f(1) = a + b$$

$$\text{So } 4a + b = 3(a + b)$$

$$4a + b = 3a + 3b$$

$$a = 2b$$

$$f(2) = 10 \text{ means } 2a + b = 10.$$

$$\text{Substitute } a = 2b: 4b + b = 10, \text{ so } 5b = 10$$

$$b = 2 \quad \text{and} \quad a = 2 \times 2 = 4$$

$$\text{Check: } f(4) = 18 \text{ and } 3 \times f(1) = 3 \times 6 = 18 \quad \checkmark$$

Answer: $a = 4, b = 2$

Tip: $3 \times f(1)$ is $3(a + b)$, not $3a + b$. Use a bracket.

Marks: 1 mark for $4a + b = 3(a + b)$ or $2a + b = 10$. 1 mark for $a = 2b$ (or another correct equation in one letter). 1 mark for $a = 4$ and $b = 2$.

7 (2 marks)

$$f(x) = x^2$$

Olu says,

" $f(2x)$ is always equal to $2f(x)$ "

Is Olu correct?

Show working to support your answer.

$$f(2x) \text{ means replace } x \text{ with } 2x: f(2x) = (2x)^2 = 4x^2$$

$$2f(x) = 2x^2$$

$$4x^2 = 2x^2 \text{ only when } x = 0.$$

$$\text{For example, } f(2 \times 3) = 36 \text{ but } 2f(3) = 18.$$

So Olu is not correct.

Answer: No, $f(2x) = 4x^2$ but $2f(x) = 2x^2$

Tip: $(2x)^2$ means $2x \times 2x = 4x^2$. Squaring only the x to get $2x^2$ is a common mistake.

Marks: 1 mark for $4x^2$ (or a correct numerical example). 1 mark for comparing with $2x^2$ and saying No.

8 (2 marks)

$$f(x) = 5 - 2x$$

Nia says,

"The equation $f(x) = f(x + 3)$ has no solutions."

Is Nia correct?

Show working to support your answer.

$$f(x + 3) = 5 - 2(x + 3) = 5 - 2x - 6 = -1 - 2x$$

$$\text{The equation is } 5 - 2x = -1 - 2x$$

$$\text{Add } 2x \text{ to both sides: } 5 = -1$$

This is never true, so there are no solutions.

So Nia is correct.

Answer: Yes, it simplifies to $5 = -1$, which is never true

Tip: Put $(x + 3)$ in a bracket when you substitute: $5 - 2(x + 3)$, not $5 - 2x + 3$.

Marks: 1 mark for $f(x + 3) = -1 - 2x$ (or $5 - 2(x + 3)$). 1 mark for showing the x terms cancel to give a false statement, with Yes.

9 (3 marks)

$f(x) = 2x - 1$ and $g(x) = x^2$
Expand and simplify $g(x - 3) - f(2x)$

$g(x - 3)$: replace x with $(x - 3)$ in x^2 . This gives $(x - 3)^2 = x^2 - 6x + 9$

$f(2x)$: replace x with $2x$ in $2x - 1$. This gives $4x - 1$

Subtract: $x^2 - 6x + 9 - (4x - 1)$

$$= x^2 - 6x + 9 - 4x + 1$$

$$= x^2 - 10x + 10$$

Answer: $x^2 - 10x + 10$

Tip: Put $f(2x)$ in a bracket before you subtract it. $-(4x - 1)$ becomes $-4x + 1$.

Marks: 1 mark for $x^2 - 6x + 9$. 1 mark for $4x - 1$. 1 mark for $x^2 - 10x + 10$.

10 (3 marks)

$f(x) = x + 2$ and $g(x) = 3x + 10$
Solve $(f(x))^2 = g(x)$

$$(f(x))^2 = (x + 2)^2$$

$$\text{So } (x + 2)^2 = 3x + 10$$

$$\text{Expand: } x^2 + 4x + 4 = 3x + 10$$

$$\text{Rearrange: } x^2 + x - 6 = 0$$

$$\text{Factorise: } (x + 3)(x - 2) = 0$$

So $x = -3$ or $x = 2$

Answer: $x = 2$ or $x = -3$

Marks: 1 mark for $(x + 2)^2 = 3x + 10$. 1 mark for $x^2 + x - 6 = 0$. 1 mark for both answers.

11 (3 marks)

$$h(x) = 3 \times 2^x$$

(a) Work out $h(-1)$

$$h(-1) = 3 \times 2^{-1}$$

$$2^{-1} = \frac{1}{2}$$

$$3 \times \frac{1}{2} = \frac{3}{2}$$

Answer: $\frac{3}{2}$ (or 1.5)

Tip: A negative power means a reciprocal: $2^{-1} = \frac{1}{2}$, not -2 .

(b) Solve $h(x) = 96$

$$3 \times 2^x = 96$$

Divide both sides by 3: $2^x = 32$

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

So $x = 5$

Answer: $x = 5$

Tip: Divide by 3 first. 3×2^x is not the same as 6^x .

Marks: 1 mark for $2^x = 32$. 1 mark for $x = 5$.
