

Factorising Harder Quadratics

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Fill in the boxes to complete each factorisation.

(a) $3x^2 + 7x + 2 = (3x + \boxed{})(x + \boxed{})$

The two numbers in the boxes must multiply to give 2.

So they are 1 and 2, in some order.

Try $(3x + 1)(x + 2)$:

$$3x^2 + 6x + x + 2 = 3x^2 + 7x + 2 \quad \checkmark$$

Answer: $(3x + 1)(x + 2)$

(b) $3x^2 - 7x - 6 = (3x + \boxed{})(x - \boxed{})$

The two numbers in the boxes must multiply to give 6.

Try the pairs 1, 6 and 2, 3 in both orders.

Try $(3x + 2)(x - 3)$:

$$3x^2 - 9x + 2x - 6 = 3x^2 - 7x - 6 \quad \checkmark$$

Answer: $(3x + 2)(x - 3)$

2 (6 marks)

Factorise

(a) $2x^2 + 11x + 5$

Multiply the first and last numbers: $2 \times 5 = 10$

Find two numbers that multiply to 10 and add to 11: 10 and 1

Split the middle term: $2x^2 + 10x + x + 5$

Factorise in pairs: $2x(x + 5) + 1(x + 5)$

Both parts have $(x + 5)$, so take it out.

Answer: $(2x + 1)(x + 5)$

Marks: 1 mark for $(2x \pm 1)(x \pm 5)$. 1 mark for the correct answer.

(b) $3x^2 - 10x + 8$

Multiply the first and last numbers: $3 \times 8 = 24$

Find two numbers that multiply to 24 and add to -10 : -6 and -4

Split the middle term: $3x^2 - 6x - 4x + 8$

Factorise in pairs: $3x(x - 2) - 4(x - 2)$

Take out $(x - 2)$.

Answer: $(3x - 4)(x - 2)$

Marks: 1 mark for $(3x \pm 4)(x \pm 2)$. 1 mark for the correct answer.

(c) $6x^2 + x - 2$

Multiply the first and last numbers: $6 \times (-2) = -12$

Find two numbers that multiply to -12 and add to 1: 4 and -3

Split the middle term: $6x^2 + 4x - 3x - 2$

Factorise in pairs: $2x(3x + 2) - 1(3x + 2)$

Take out $(3x + 2)$.

Check: $(3x + 2)(2x - 1) = 6x^2 - 3x + 4x - 2 = 6x^2 + x - 2$ ✓

Answer: $(3x + 2)(2x - 1)$

Tip: Brackets with the wrong signs, like $(3x - 2)(2x + 1)$, give the wrong middle term. Always expand your answer to check it.

Marks: 1 mark for brackets that give two of the three terms. 1 mark for the correct answer.

3 (4 marks)

Factorise fully

(a) $4x^2 - 25$

$$4x^2 = (2x)^2 \text{ and } 25 = 5^2$$

So this is a difference of two squares: $(2x)^2 - 5^2$

Use $a^2 - b^2 = (a + b)(a - b)$.

Answer: $(2x + 5)(2x - 5)$

(b) $12x^2 - 27$

First take out the common factor 3:

$$12x^2 - 27 = 3(4x^2 - 9)$$

$4x^2 - 9 = (2x)^2 - 3^2$ is a difference of two squares.

$$\text{So } 4x^2 - 9 = (2x + 3)(2x - 3)$$

Answer: $3(2x + 3)(2x - 3)$

Tip: "Factorise fully" here needs two steps: take out the common factor 3 **and** use the difference of two squares.

Marks: 1 mark for $3(4x^2 - 9)$ or $(6x + 9)(2x - 3)$. 1 mark for the correct answer.

(c) $9x^2 - 16y^2$

$$9x^2 = (3x)^2 \text{ and } 16y^2 = (4y)^2$$

So this is $(3x)^2 - (4y)^2$, a difference of two squares.

Answer: $(3x + 4y)(3x - 4y)$

4 (1 mark)

Circle the factorisation of $2x^2 - x - 15$

Options: $(2x - 5)(x + 3)$, $(2x + 5)(x - 3)$, $(2x - 3)(x + 5)$, $(2x + 3)(x - 5)$

Expand each option and look for a middle term of $-x$.

$$(2x - 5)(x + 3) = 2x^2 + 6x - 5x - 15 = 2x^2 + x - 15$$

$$(2x + 5)(x - 3) = 2x^2 - 6x + 5x - 15 = 2x^2 - x - 15 \quad \checkmark$$

Answer: $(2x + 5)(x - 3)$

5 (3 marks)

Solve $3x^2 + 5x - 2 = 0$

Factorise the left side.

$3 \times (-2) = -6$, and 6 and -1 multiply to -6 and add to 5.

$$3x^2 + 6x - x - 2 = 3x(x + 2) - 1(x + 2)$$

$$\text{So } (3x - 1)(x + 2) = 0$$

If two things multiply to make 0, one of them must be 0.

$$3x - 1 = 0, \text{ so } x = \frac{1}{3}$$

$$x + 2 = 0, \text{ so } x = -2$$

Answer: $x = \frac{1}{3}$ or $x = -2$

Marks: 1 mark for $(3x \pm 1)(x \pm 2)$. 1 mark for $(3x - 1)(x + 2)$. 1 mark for both solutions.

6 (2 marks)

Factorise $6x^2 - 11x - 35$

Multiply the first and last numbers: $6 \times (-35) = -210$

Find two numbers that multiply to -210 and add to -11 : -21 and 10

Split the middle term: $6x^2 - 21x + 10x - 35$

Factorise in pairs: $3x(2x - 7) + 5(2x - 7)$

Take out $(2x - 7)$.

Answer: $(2x - 7)(3x + 5)$

Tip: Your answer must be two brackets multiplied together. $x(6x - 11) - 35$ is not factorised, and gets no marks.

Marks: 1 mark for brackets that give two of the three terms. 1 mark for the correct answer.

7 (4 marks)

Factorise fully

(a) $x^3 - 36x$

Take out the common factor x :

$$x^3 - 36x = x(x^2 - 36)$$

$x^2 - 36 = x^2 - 6^2$ is a difference of two squares.

$$\text{So } x^2 - 36 = (x + 6)(x - 6)$$

Answer: $x(x + 6)(x - 6)$

Marks: 1 mark for $x(x^2 - 36)$. 1 mark for the correct answer.

(b) $4x^2 - 14x - 30$

All the numbers are even, so take out the common factor 2:

$$4x^2 - 14x - 30 = 2(2x^2 - 7x - 15)$$

Now factorise $2x^2 - 7x - 15$.

$2 \times (-15) = -30$, and -10 and 3 multiply to -30 and add to -7 .

$$2x^2 - 10x + 3x - 15 = 2x(x - 5) + 3(x - 5) = (2x + 3)(x - 5)$$

Remember to keep the 2 at the front.

Answer: $2(2x + 3)(x - 5)$

Tip: This is an expression, not an equation, so you cannot divide the 2 away. It must stay in your answer.

Marks: 1 mark for $2(2x^2 - 7x - 15)$ or $(2x + 3)(2x - 10)$. 1 mark for the correct answer.

8 (2 marks)

A rectangle has area $(6x^2 + 13x + 6) \text{ cm}^2$.

The width of the rectangle is $(2x + 3) \text{ cm}$.

Work out an expression for the length.

Area = width $\times$ length

So $(2x + 3)$ must be one of the factors of $6x^2 + 13x + 6$.

$6x^2 = 2x \times 3x$ and $6 = 3 \times 2$, so try $(2x + 3)(3x + 2)$:

$$(2x + 3)(3x + 2) = 6x^2 + 4x + 9x + 6 = 6x^2 + 13x + 6 \quad \checkmark$$

So the length is $3x + 2$.

Answer: $(3x + 2) \text{ cm}$

Marks: 1 mark for $(2x + 3)(3x + 2)$. 1 mark for the correct answer.

9 (3 marks)

(a) Factorise fully $3n^2 + 10n + 8$

Multiply the first and last numbers: $3 \times 8 = 24$

Find two numbers that multiply to 24 and add to 10: 6 and 4

Split the middle term: $3n^2 + 6n + 4n + 8$

Factorise in pairs: $3n(n + 2) + 4(n + 2)$

Take out $(n + 2)$.

Answer: $(3n + 4)(n + 2)$

Marks: 1 mark for $(3n \pm 4)(n \pm 2)$. 1 mark for the correct answer.

- (b) n is an odd number. Is $3n^2 + 10n + 8$ always odd, always even, or could it be either?

From part (a), $3n^2 + 10n + 8 = (3n + 4)(n + 2)$.

n is odd, so $3n$ is odd, and odd + 4 is odd. So $3n + 4$ is odd.

n is odd, so $n + 2$ is odd.

odd $\times$ odd = odd, so the expression is always odd.

For example, $n = 1$ gives $7 \times 3 = 21$, and $n = 3$ gives $13 \times 5 = 65$.

Answer: Always odd: $(3n + 4)$ and $(n + 2)$ are both odd when n is odd, and odd $\times$ odd is odd

Tip: Use the factorised form from part (a). Trying one or two values of n is not a reason: explain why it works for **every** odd n .