

Expanding Double and Triple Brackets

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Expand and simplify

(a) $(x + 3)(x - 5)$

Multiply each term in the first bracket by each term in the second:

$$x \times x = x^2$$

$$x \times (-5) = -5x$$

$$3 \times x = 3x$$

$$3 \times (-5) = -15$$

$$\text{So } (x + 3)(x - 5) = x^2 - 5x + 3x - 15$$

$$\text{Collect the } x \text{ terms: } -5x + 3x = -2x$$

Answer: $x^2 - 2x - 15$

Marks: 1 mark for four terms with at least three correct. 1 mark for the correct answer.

(b) $(2x - 1)(3x + 4)$

$$2x \times 3x = 6x^2$$

$$2x \times 4 = 8x$$

$$-1 \times 3x = -3x$$

$$-1 \times 4 = -4$$

$$\text{So } (2x - 1)(3x + 4) = 6x^2 + 8x - 3x - 4$$

Collect the x terms: $8x - 3x = 5x$

Answer: $6x^2 + 5x - 4$

Marks: 1 mark for four terms with at least three correct. 1 mark for the correct answer.

2 (1 mark)

Circle the expansion of $(x - 4)^2$

Options: $x^2 + 16$, $x^2 - 16$, $x^2 - 8x + 16$, $x^2 - 8x - 16$

$(x - 4)^2$ means $(x - 4)(x - 4)$.

$$= x^2 - 4x - 4x + 16$$

$$= x^2 - 8x + 16$$

Answer: $x^2 - 8x + 16$

Tip: $(x - 4)^2$ is **not** $x^2 - 16$ or $x^2 + 16$. Always write out the two brackets so you do not miss the middle term.

3 (6 marks)

Expand and simplify

(a) $(x + 1)(x + 3)(x + 4)$

Expand the first two brackets:

$$(x + 1)(x + 3) = x^2 + 3x + x + 3 = x^2 + 4x + 3$$

Now multiply by the third bracket:

$$(x^2 + 4x + 3)(x + 4)$$

$$x^2 \times x = x^3 \quad x^2 \times 4 = 4x^2$$

$$4x \times x = 4x^2 \quad 4x \times 4 = 16x$$

$$3 \times x = 3x \quad 3 \times 4 = 12$$

$$\text{So we get } x^3 + 4x^2 + 4x^2 + 16x + 3x + 12$$

Collect like terms.

Answer: $x^3 + 8x^2 + 19x + 12$

Tip: Multiply two brackets first and simplify. Then multiply by the third bracket. A three-term bracket times a two-term bracket gives $3 \times 2 = 6$ terms, so check you have six.

Marks: 1 mark for expanding two brackets correctly. 1 mark for six terms from multiplying by the third bracket. 1 mark for the correct answer.

(b) $(2x + 1)^2(x - 3)$

First expand the square:

$$(2x + 1)^2 = (2x + 1)(2x + 1) = 4x^2 + 2x + 2x + 1 = 4x^2 + 4x + 1$$

Now multiply by $(x - 3)$:

$$4x^2 \times x = 4x^3 \quad 4x^2 \times (-3) = -12x^2$$

$$4x \times x = 4x^2 \quad 4x \times (-3) = -12x$$

$$1 \times x = x \quad 1 \times (-3) = -3$$

$$\text{So we get } 4x^3 - 12x^2 + 4x^2 - 12x + x - 3$$

Collect like terms.

Answer: $4x^3 - 8x^2 - 11x - 3$

Tip: $(2x + 1)^2$ is not $4x^2 + 1$. Write it as $(2x + 1)(2x + 1)$ first.

Marks: 1 mark for $4x^2 + 4x + 1$. 1 mark for six terms. 1 mark for the correct answer.

4 (2 marks)

$$(x + a)(x + 5) \equiv x^2 + bx + 15$$

Work out the values of a and b .

$$\text{Expand the left side: } (x + a)(x + 5) = x^2 + 5x + ax + 5a$$

$$\text{So } x^2 + (5 + a)x + 5a \equiv x^2 + bx + 15$$

$$\text{Compare the number terms: } 5a = 15, \text{ so } a = 3$$

Compare the x terms: $b = 5 + a = 5 + 3 = 8$

Answer: $a = 3$, $b = 8$

Marks: 1 mark for $a = 3$. 1 mark for $b = 8$.

5 (4 marks)

Expand and simplify $(3x^2 - 1)(x + 4) - 3x(x^2 + 2)$

Expand the first product:

$$(3x^2 - 1)(x + 4) = 3x^3 + 12x^2 - x - 4$$

Expand the second product:

$$3x(x^2 + 2) = 3x^3 + 6x$$

Subtract the whole of the second product (use a bracket):

$$3x^3 + 12x^2 - x - 4 - (3x^3 + 6x)$$

$$= 3x^3 + 12x^2 - x - 4 - 3x^3 - 6x$$

The x^3 terms cancel: $3x^3 - 3x^3 = 0$

$$-x - 6x = -7x$$

Answer: $12x^2 - 7x - 4$

Tip: The minus sign in front of $3x$ applies to **both** terms: $-3x(x^2 + 2) = -3x^3 - 6x$.

Marks: 1 mark for four terms for the first product. 1 mark for $3x^3 + 12x^2 - x - 4$. 1 mark for subtracting $3x^3 + 6x$. 1 mark for the correct answer.

6 (4 marks)

Expand and simplify $(x - 3)(2x + y)^2$

First expand the square:

$$(2x + y)^2 = (2x + y)(2x + y)$$

$$= 4x^2 + 2xy + 2xy + y^2$$

$$= 4x^2 + 4xy + y^2$$

Now multiply by $(x - 3)$:

$$x \times (4x^2 + 4xy + y^2) = 4x^3 + 4x^2y + xy^2$$

$$-3 \times (4x^2 + 4xy + y^2) = -12x^2 - 12xy - 3y^2$$

There are no like terms to collect.

Answer: $4x^3 + 4x^2y + xy^2 - 12x^2 - 12xy - 3y^2$

Tip: A common wrong answer is $(2x + y)^2 = 4x^2 + y^2$. The middle terms $2xy + 2xy = 4xy$ must be there.

Marks: 1 mark for $(2x + y)^2$ expanded with at least three terms correct. 1 mark for $4x^2 + 4xy + y^2$. 1 mark for six terms. 1 mark for the correct answer.

7 (3 marks)

A cuboid has length $(x + 4)$ cm, width $(x - 1)$ cm and height $(2x + 3)$ cm.

Show that the volume, in cm^3 , is $2x^3 + 9x^2 + x - 12$

$$\text{Volume} = \text{length} \times \text{width} \times \text{height} = (x + 4)(x - 1)(2x + 3)$$

Expand the first two brackets:

$$(x + 4)(x - 1) = x^2 - x + 4x - 4 = x^2 + 3x - 4$$

Now multiply by $(2x + 3)$:

$$(x^2 + 3x - 4)(2x + 3) = 2x^3 + 3x^2 + 6x^2 + 9x - 8x - 12$$

Collect like terms:

$$= 2x^3 + 9x^2 + x - 12 \text{ as required.}$$

Answer: Shown

Marks: 1 mark for two brackets expanded correctly. 1 mark for six terms. 1 mark for fully correct working.

8 (5 marks)

- (a) Expand and simplify fully $(x + 3)(2x + 1)(x + 4)$

You can multiply the brackets in any order.

Expand $(x + 3)(x + 4)$ first:

$$(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$$

Now multiply by $(2x + 1)$:

$$(x^2 + 7x + 12)(2x + 1) = 2x^3 + x^2 + 14x^2 + 7x + 24x + 12$$

Collect like terms.

Answer: $2x^3 + 15x^2 + 31x + 12$

Marks: 1 mark for two brackets expanded correctly. 1 mark for six terms. 1 mark for the correct answer.

- (b) Use your answer from part (a) to work out $103 \times 201 \times 104$

Put $x = 100$:

$$x + 3 = 103, \quad 2x + 1 = 201, \quad x + 4 = 104$$

So $103 \times 201 \times 104$ is the answer to part (a) when $x = 100$.

$$2 \times 100^3 = 2\,000\,000$$

$$15 \times 100^2 = 150\,000$$

$$31 \times 100 = 3100$$

$$\text{Add: } 2\,000\,000 + 150\,000 + 3100 + 12 = 2\,153\,112$$

Answer: 2 153 112

Tip: “Use your answer” means use part (a). Spot which value of x turns the brackets into 103, 201 and 104. Do not use long multiplication.

Marks: 1 mark for substituting $x = 100$ into your expansion. 1 mark for the correct answer (follow-through from part (a)).