

Exact Trig Values - Higher

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the value of $\tan 60^\circ$

Options: $\frac{\sqrt{3}}{3}$, $\frac{\sqrt{3}}{2}$, 1, $\sqrt{3}$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Answer: $\sqrt{3}$

Tip: $\frac{\sqrt{3}}{2}$ is $\sin 60^\circ$ (and $\cos 30^\circ$), not $\tan 60^\circ$. $\tan 60^\circ$ is bigger than 1, because the opposite side is longer than the adjacent side.

2 (4 marks)

Work out the exact value of

(a) $6 \cos 30^\circ$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$6 \times \frac{\sqrt{3}}{2} = \frac{6\sqrt{3}}{2}$$

$$= 3\sqrt{3}$$

Answer: $3\sqrt{3}$

Tip: Only the top of the fraction is multiplied: $6 \times \frac{\sqrt{3}}{2} = \frac{6\sqrt{3}}{2}$, not $\frac{6\sqrt{3}}{12}$.

Marks: 1 mark for $\cos 30^\circ = \frac{\sqrt{3}}{2}$. 1 mark for $3\sqrt{3}$.

(b) $\sin 45^\circ \times \cos 45^\circ$

$$\sin 45^\circ = \frac{\sqrt{2}}{2} \text{ and } \cos 45^\circ = \frac{\sqrt{2}}{2}$$

Multiply the tops and multiply the bottoms:

$$\frac{\sqrt{2} \times \sqrt{2}}{2 \times 2} = \frac{2}{4}$$
$$= \frac{1}{2}$$

Answer: $\frac{1}{2}$

Tip: $\sqrt{2} \times \sqrt{2} = 2$, not 4. Multiplying a square root by itself gives the number under the root.

Marks: 1 mark for both values. 1 mark for $\frac{1}{2}$.

3 (2 marks)

Write $\frac{3}{\sin 60^\circ}$ in the form $a\sqrt{3}$, where a is an integer.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$3 \div \frac{\sqrt{3}}{2} = 3 \times \frac{2}{\sqrt{3}} = \frac{6}{\sqrt{3}}$$

Rationalise: multiply the top and bottom by $\sqrt{3}$

$$\frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6\sqrt{3}}{3}$$
$$= 2\sqrt{3}$$

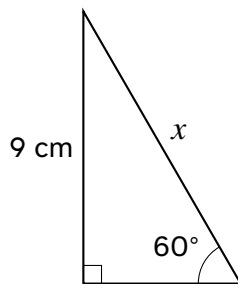
Answer: $2\sqrt{3}$

Tip: To divide by a fraction, multiply by its reciprocal. Then rationalise: you can't just remove the root sign from the bottom.

Marks: 1 mark for $\frac{6}{\sqrt{3}}$. 1 mark for $2\sqrt{3}$.

4 (3 marks)

Here is a right-angled triangle.



Not drawn accurately

Work out the length x .

Give your answer in the form $a\sqrt{3}$

From the 60° angle: 9 cm is the opposite and x is the hypotenuse.

$$\text{SOH: } \sin 60^\circ = \frac{9}{x}$$

$$x = \frac{9}{\sin 60^\circ} = 9 \div \frac{\sqrt{3}}{2} = \frac{18}{\sqrt{3}}$$

$$\text{Rationalise: } \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{18\sqrt{3}}{3}$$

$$= 6\sqrt{3}$$

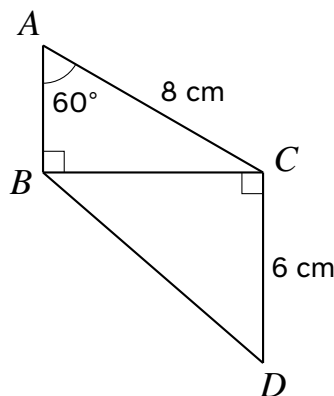
Answer: $6\sqrt{3}$ cm

Tip: Check your answer: $6\sqrt{3}$ is about 10.4, which is more than 9. The hypotenuse must be the longest side, so $9 \times \sin 60^\circ$ can't be right.

Marks: 1 mark for $\sin 60^\circ = \frac{9}{x}$ (or $x = \frac{9}{\sin 60^\circ}$). 1 mark for $\frac{18}{\sqrt{3}}$. 1 mark for $6\sqrt{3}$.

5 (4 marks)

ABC and BCD are right-angled triangles.



Not drawn accurately

$AC = 8$ cm and $CD = 6$ cm

Work out the length of BD .

Give your answer in the form $a\sqrt{b}$, where a and b are integers greater than 1

In triangle ABC : from the 60° angle, BC is the opposite and AC is the hypotenuse.

$$BC = 8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

In triangle BCD , use Pythagoras: $BD^2 = BC^2 + CD^2$

$$(4\sqrt{3})^2 = 16 \times 3 = 48$$

$$BD^2 = 48 + 36 = 84$$

$$BD = \sqrt{84} = \sqrt{4} \times \sqrt{21} = 2\sqrt{21}$$

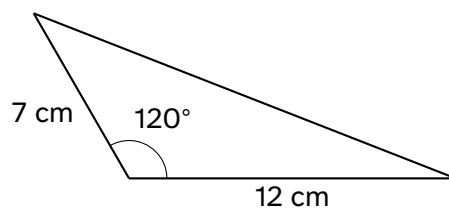
Answer: $2\sqrt{21}$ cm

Tip: $(4\sqrt{3})^2$ is $16 \times 3 = 48$. Square the 4 as well as the $\sqrt{3}$: it is not $4 \times 3 = 12$.

Marks: 1 mark for $8 \sin 60^\circ$. 1 mark for $BC = 4\sqrt{3}$. 1 mark for $BD^2 = 48 + 36$ or $\sqrt{84}$. 1 mark for $2\sqrt{21}$.

6 (3 marks)

Here is a triangle.



Not drawn accurately

Show that the area of the triangle is $21\sqrt{3} \text{ cm}^2$

The 120° angle is between the 12 cm and 7 cm sides, so use $\text{area} = \frac{1}{2}ab \sin C$

$$\text{Area} = \frac{1}{2} \times 12 \times 7 \times \sin 120^\circ = 42 \sin 120^\circ$$

$$\text{From the graph of } y = \sin x: \sin 120^\circ = \sin(180^\circ - 120^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Area} = 42 \times \frac{\sqrt{3}}{2} = \frac{42\sqrt{3}}{2}$$

$$= 21\sqrt{3} \text{ cm}^2 \quad \checkmark$$

Answer: $\text{Area} = 42 \times \frac{\sqrt{3}}{2} = 21\sqrt{3}$

Tip: You can't use $\frac{1}{2} \times \text{base} \times \text{height}$ with 12 and 7, because they are not perpendicular. Use $\frac{1}{2}ab \sin C$, with the angle between the two sides.

Marks: 1 mark for $\frac{1}{2} \times 12 \times 7 \times \sin 120^\circ$. 1 mark for $\sin 120^\circ = \frac{\sqrt{3}}{2}$. 1 mark for showing it equals $21\sqrt{3}$.

7 (3 marks)

Work out the exact value of

$$(\tan 30^\circ + \tan 60^\circ) \times \sin 45^\circ$$

Give your answer in the form $\frac{a\sqrt{b}}{c}$, where a , b and c are integers.

$$\tan 30^\circ = \frac{\sqrt{3}}{3}, \quad \tan 60^\circ = \sqrt{3} \quad \text{and} \quad \sin 45^\circ = \frac{\sqrt{2}}{2}$$

Work out the bracket first. Write $\sqrt{3}$ as $\frac{3\sqrt{3}}{3}$:

$$\frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$$

Now multiply: $\frac{4\sqrt{3}}{3} \times \frac{\sqrt{2}}{2} = \frac{4\sqrt{6}}{6}$

Simplify (divide the top and bottom by 2): $\frac{2\sqrt{6}}{3}$

Answer: $\frac{2\sqrt{6}}{3}$

Tip: When you add fractions, the denominators are not added: $\frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3}$ is $\frac{4\sqrt{3}}{3}$, not $\frac{4\sqrt{3}}{6}$.
Examiners see this mistake every year.

Marks: 1 mark for all three values. 1 mark for $\frac{4\sqrt{3}}{3}$ (or $\frac{4}{\sqrt{3}}$). 1 mark for $\frac{2\sqrt{6}}{3}$ (or $\frac{4\sqrt{6}}{6}$).

8 (3 marks)

y is an acute angle.

$$4 \cos y \times \tan 60^\circ = 6 \tan 45^\circ$$

Work out the value of y .

You must show your working.

$$\tan 60^\circ = \sqrt{3} \text{ and } \tan 45^\circ = 1$$

$$\text{So } 4\sqrt{3} \cos y = 6$$

$$\cos y = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

$$\text{Rationalise: } \frac{3 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \text{ so } y = 30^\circ$$

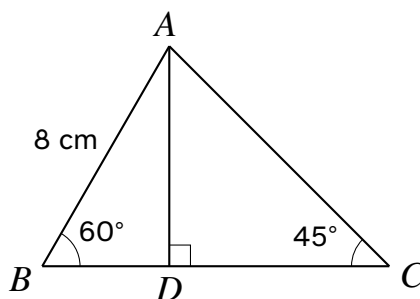
Answer: 30°

Tip: The question asks for the angle, so don't stop at $\cos y = \frac{\sqrt{3}}{2}$. Use the exact values backwards to find y .

Marks: 1 mark for $4\sqrt{3} \cos y = 6$. 1 mark for $\cos y = \frac{\sqrt{3}}{2}$ (or $\frac{3}{2\sqrt{3}}$). 1 mark for 30.

9 (5 marks)

Here is triangle ABC .



Not drawn accurately

D is a point on BC and AD is perpendicular to BC .

$AB = 8$ cm, angle $ABC = 60^\circ$ and angle $ACB = 45^\circ$

Work out the area of triangle ABC .

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

$$\text{In triangle } ABD: AD = 8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$$BD = 8 \cos 60^\circ = 8 \times \frac{1}{2} = 4$$

$$\text{In triangle } ADC: \tan 45^\circ = \frac{AD}{DC} = 1, \text{ so } DC = AD = 4\sqrt{3}$$

$$BC = BD + DC = 4 + 4\sqrt{3}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (4 + 4\sqrt{3}) \times 4\sqrt{3}$$

$$= 2\sqrt{3} \times (4 + 4\sqrt{3})$$

$$= 8\sqrt{3} + 8 \times 3$$

$$= 24 + 8\sqrt{3}$$

Answer: $24 + 8\sqrt{3} \text{ cm}^2$

Tip: Triangle ADC has angles of 45° and 90° , so it is isosceles and $DC = AD$. When you multiply out, $2\sqrt{3} \times 4\sqrt{3} = 8 \times 3 = 24$.

Marks: 1 mark for $AD = 4\sqrt{3}$. 1 mark for $BD = 4$. 1 mark for $DC = 4\sqrt{3}$. 1 mark for $\frac{1}{2} \times (4 + 4\sqrt{3}) \times 4\sqrt{3}$. 1 mark for $24 + 8\sqrt{3}$.
