

Equations with Algebraic Fractions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Solve $\frac{5}{x+1} = 2$

Multiply both sides by $(x + 1)$:

$$5 = 2(x + 1)$$

$$5 = 2x + 2$$

Subtract 2 from both sides: $3 = 2x$

Divide both sides by 2: $x = \frac{3}{2}$

Answer: $\frac{3}{2}$

Marks: 1 mark for $5 = 2(x + 1)$. 1 mark for $\frac{3}{2}$ (or 1.5).

2 (3 marks)

Solve $\frac{12}{x-2} = x + 2$

Multiply both sides by $(x - 2)$:

$$12 = (x + 2)(x - 2)$$

Expand (difference of two squares): $12 = x^2 - 4$

Add 4 to both sides: $x^2 = 16$

Square root, remembering the $\pm$: $x = \pm 4$

Answer: $x = 4$ or $x = -4$

Marks: 1 mark for multiplying by $(x - 2)$. 1 mark for $x^2 = 16$. 1 mark for both answers.

3 (4 marks)

Solve $\frac{3}{x} + \frac{2}{x-1} = 2$

Multiply every term by $x(x - 1)$ to clear the fractions:

$$3(x - 1) + 2x = 2x(x - 1)$$

Expand: $3x - 3 + 2x = 2x^2 - 2x$

Simplify: $5x - 3 = 2x^2 - 2x$

Rearrange so one side is 0: $2x^2 - 7x + 3 = 0$

Factorise: $(2x - 1)(x - 3) = 0$

So $2x - 1 = 0$ or $x - 3 = 0$

Answer: $x = \frac{1}{2}$ or $x = 3$

Tip: Each numerator gets multiplied by the **other** denominator: $\frac{3}{x} \times x(x - 1) = 3(x - 1)$.
Remember to multiply the 2 on the right as well.

Marks: 1 mark for multiplying by $x(x - 1)$. 1 mark for $2x^2 - 7x + 3 = 0$. 1 mark for factorising (or using the formula). 1 mark for both answers.

4 (5 marks)

(a) Show that $\frac{4}{x-3} + \frac{1}{x} = 1$ can be written as $x^2 - 8x + 3 = 0$

Multiply every term by $x(x - 3)$:

$$4x + (x - 3) = x(x - 3)$$

Simplify the left: $5x - 3$

Expand the right: $x^2 - 3x$

$$\text{So } 5x - 3 = x^2 - 3x$$

Subtract $5x$ and add 3 on both sides:

$$0 = x^2 - 8x + 3$$

So $x^2 - 8x + 3 = 0$, as required.

Answer: Shown

Marks: 1 mark for multiplying by $x(x - 3)$. 1 mark for expanding correctly. 1 mark for a fully correct method reaching the given equation.

- (b) Hence solve $\frac{4}{x-3} + \frac{1}{x} = 1$
Give your answers to 2 decimal places.

“Hence” means use part (a): $x^2 - 8x + 3 = 0$

This does not factorise, so use the quadratic formula with $a = 1$, $b = -8$, $c = 3$:

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$x = \frac{8 \pm \sqrt{52}}{2}$$

$$x = 7.605 \dots \text{ or } x = 0.394 \dots$$

Answer: $x = 7.61$ or $x = 0.39$

Tip: Give the answers in the form asked for. Here that is 2 decimal places, not a surd.

Marks: 1 mark for substituting correctly into the formula. 1 mark for both answers to 2 decimal places.

5 (4 marks)

Solve $\frac{2x+3}{x-1} = \frac{2x-1}{x+2}$
You must show your working.

Multiply both sides by $(x-1)(x+2)$ (cross-multiply):

$$(2x+3)(x+2) = (2x-1)(x-1)$$

$$\text{Expand the left: } 2x^2 + 4x + 3x + 6 = 2x^2 + 7x + 6$$

Expand the right: $2x^2 - 2x - x + 1 = 2x^2 - 3x + 1$

So $2x^2 + 7x + 6 = 2x^2 - 3x + 1$

The $2x^2$ terms cancel: $7x + 6 = -3x + 1$

$$10x = -5$$

$$x = -\frac{1}{2}$$

Check: $\frac{2}{-1.5} = -\frac{4}{3}$ and $\frac{-2}{1.5} = -\frac{4}{3}$ ✓

Answer: $-\frac{1}{2}$

Tip: Multiply each numerator by the *other* fraction's denominator. Use brackets so every term is multiplied.

Marks: 1 mark for $(2x + 3)(x + 2) = (2x - 1)(x - 1)$. 1 mark for expanding both sides correctly. 1 mark for $10x = -5$. 1 mark for $-\frac{1}{2}$.

6 (5 marks)

Solve $\frac{x}{x+2} - \frac{1}{x} = \frac{1}{3}$

Give your solutions to 2 decimal places.

You must show your working.

Multiply every term by $3x(x + 2)$:

$$\frac{x}{x+2} \times 3x(x+2) = 3x^2$$

$$\frac{1}{x} \times 3x(x+2) = 3(x+2)$$

$$\frac{1}{3} \times 3x(x+2) = x(x+2)$$

$$\text{So } 3x^2 - 3(x+2) = x(x+2)$$

$$\text{Expand: } 3x^2 - 3x - 6 = x^2 + 2x$$

$$\text{Rearrange: } 2x^2 - 5x - 6 = 0$$

Use the formula with $a = 2$, $b = -5$, $c = -6$:

$$x = \frac{5 \pm \sqrt{25 + 48}}{4} = \frac{5 \pm \sqrt{73}}{4}$$

$$x = 3.386 \dots \text{ or } x = -0.886 \dots$$

Answer: $x = 3.39$ or $x = -0.89$

Tip: Clear all the fractions in one step at the start. Keeping the denominators for longer leads to mistakes.

Marks: 1 mark for multiplying by $3x(x+2)$. 1 mark for expanding correctly. 1 mark for $2x^2 - 5x - 6 = 0$. 1 mark for using the formula correctly. 1 mark for both answers to 2 decimal places.

7 (5 marks)

Solve $\frac{x+2}{x-1} + \frac{3}{x} = 4$

Give your solutions to 2 decimal places.

You must show your working.

Multiply every term by $x(x-1)$:

$$\frac{x+2}{x-1} \times x(x-1) = x(x+2)$$

$$\frac{3}{x} \times x(x-1) = 3(x-1)$$

$$4 \times x(x-1) = 4x(x-1)$$

$$\text{So } x(x+2) + 3(x-1) = 4x(x-1)$$

$$\text{Expand: } x^2 + 2x + 3x - 3 = 4x^2 - 4x$$

$$x^2 + 5x - 3 = 4x^2 - 4x$$

$$\text{Rearrange: } 3x^2 - 9x + 3 = 0$$

$$\text{Divide by 3: } x^2 - 3x + 1 = 0$$

Use the formula with $a = 1$, $b = -3$, $c = 1$:

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$x = 2.618 \dots \text{ or } x = 0.381 \dots$$

Answer: $x = 2.62$ or $x = 0.38$

Tip: Remember to multiply the 4 on the right-hand side by $x(x - 1)$ too.

Marks: 1 mark for multiplying by $x(x - 1)$. 1 mark for expanding correctly. 1 mark for $3x^2 - 9x + 3 = 0$ or $x^2 - 3x + 1 = 0$. 1 mark for using the formula correctly. 1 mark for both answers to 2 decimal places.
