

Equation of a Circle and Tangents

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the radius of the circle $x^2 + y^2 = 49$

Options: 7, 24.5, 49, 98

A circle with centre (0,0) and radius r has equation $x^2 + y^2 = r^2$.

So $r^2 = 49$

$r = \sqrt{49} = 7$

Answer: 7

2 (1 mark)

A circle has centre (0,0) and diameter 10

Circle the equation of the circle.

Options: $x^2 + y^2 = 10$, $x^2 + y^2 = 20$, $x^2 + y^2 = 25$, $x^2 + y^2 = 100$

The radius is half the diameter: $10 \div 2 = 5$

A circle with centre (0,0) and radius r has equation $x^2 + y^2 = r^2$.

$r^2 = 5^2 = 25$

Answer: $x^2 + y^2 = 25$

Tip: Halve the diameter to get the radius, then **square** it. $x^2 + y^2 = 100$ uses the diameter instead of the radius, a very common mistake.

3 (1 mark)

A circle has centre $(0, 0)$ and radius 6
Write down the equation of the circle.

Use $x^2 + y^2 = r^2$ with $r = 6$.

$$r^2 = 6^2 = 36$$

Answer: $x^2 + y^2 = 36$

4 (2 marks)

A circle has centre $(0, 0)$ and circumference 20π
Work out the equation of the circle.

$$\text{Circumference} = 2\pi r$$

$$\text{So } 2\pi r = 20\pi$$

Divide both sides by 2π : $r = 10$

$$r^2 = 10^2 = 100$$

Answer: $x^2 + y^2 = 100$

Tip: The number on the right of the equation is r^2 , not r . Here it is 100, not 10 or 20.

Marks: 1 mark for $r = 10$. 1 mark for $x^2 + y^2 = 100$.

5 (2 marks)

Does the point $(5, 3)$ lie inside, outside or on the circle $x^2 + y^2 = 30$?
You must show your working.

Substitute $x = 5$ and $y = 3$ into $x^2 + y^2$:

$$5^2 + 3^2 = 25 + 9 = 34$$

$34 > 30$, so the point is further from the centre than the circle is.

So the point is outside the circle.

Answer: Outside

Marks: 1 mark for $5^2 + 3^2 = 34$. 1 mark for outside, with 34 compared to 30.

6 (3 marks)

The point $P(-5, 1)$ lies on the circle $x^2 + y^2 = 26$

Work out the equation of the tangent to the circle at P .

A tangent is perpendicular to the radius at the point where it touches the circle.

$$\text{Gradient of radius } OP = \frac{1 - 0}{-5 - 0} = -\frac{1}{5}$$

Tangent gradient = 5 (negative reciprocal of $-\frac{1}{5}$)

$$y = 5x + c$$

Substitute $P(-5, 1)$: $1 = 5 \times (-5) + c$

$$1 = -25 + c, \text{ so } c = 26$$

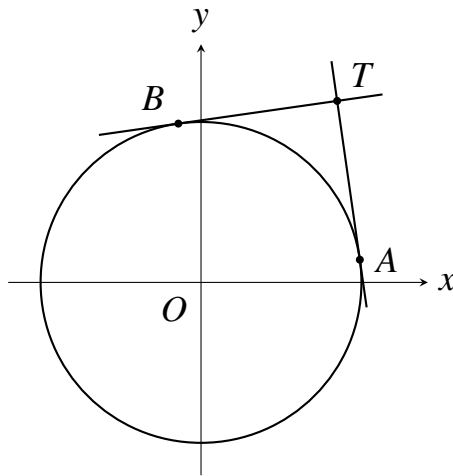
Answer: $y = 5x + 26$

Tip: Always start with the gradient of the radius OP , then take the negative reciprocal. Take care with the negative x -coordinate: $1 \div (-5) = -\frac{1}{5}$.

Marks: 1 mark for the gradient of OP . 1 mark for the tangent gradient with P substituted. 1 mark for the correct equation.

7 (5 marks)

The points $A(7, 1)$ and $B(-1, 7)$ lie on the circle $x^2 + y^2 = 50$



Not drawn accurately

The tangent at A and the tangent at B meet at the point T .

Work out the coordinates of T .

You must show your working.

Tangent at A : gradient of $OA = \frac{1}{7}$, so the tangent gradient is -7 .

$$y = -7x + c \text{ and } 1 = -7 \times 7 + c, \text{ so } c = 50$$

$$\text{Tangent at } A: y = -7x + 50$$

Tangent at B : gradient of $OB = \frac{7}{-1} = -7$, so the tangent gradient is $\frac{1}{7}$.

$$y = \frac{1}{7}x + c \text{ and } 7 = \frac{1}{7} \times (-1) + c, \text{ so } c = \frac{50}{7}$$

$$\text{Tangent at } B: y = \frac{1}{7}x + \frac{50}{7}, \text{ or } 7y = x + 50$$

Where they meet: substitute $y = -7x + 50$ into $7y = x + 50$:

$$7(-7x + 50) = x + 50$$

$$-49x + 350 = x + 50$$

$$300 = 50x, \text{ so } x = 6$$

$$y = -7 \times 6 + 50 = 8$$

Answer: (6, 8)

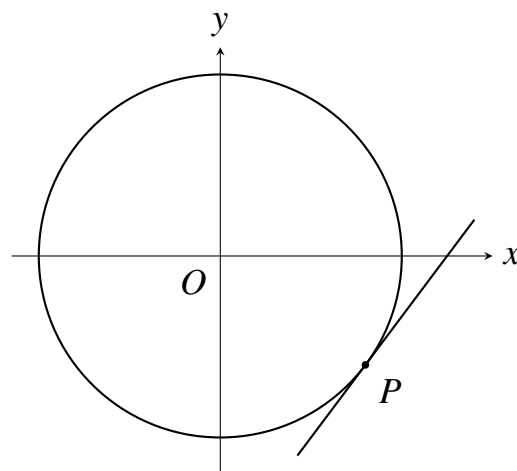
Tip: Find each tangent in the same way: gradient of the radius, then the negative reciprocal, then substitute the point. Then solve the two tangent equations simultaneously.

Marks: 1 mark for the tangent gradient -7 at A . 1 mark for $y = -7x + 50$. 1 mark for the tangent gradient $\frac{1}{7}$ at B . 1 mark for a correct equation of the tangent at B . 1 mark for (6, 8).

8 (4 marks)

The diagram shows the circle $x^2 + y^2 = 100$

P is the point (8, -6).



Not drawn accurately

Show that the line $4x - 3y = 50$ is the tangent to the circle at P .

P is on the circle: $8^2 + (-6)^2 = 64 + 36 = 100$ ✓

P is on the line: $4 \times 8 - 3 \times (-6) = 32 + 18 = 50$ ✓

Gradient of the line: $3y = 4x - 50$, so $y = \frac{4}{3}x - \frac{50}{3}$ and the gradient is $\frac{4}{3}$

Gradient of the radius OP : $\frac{-6}{8} = -\frac{3}{4}$

$\frac{4}{3} \times \left(-\frac{3}{4}\right) = -1$, so the line is perpendicular to the radius OP .

The line passes through P on the circle and is perpendicular to the radius there, so it is the tangent at P .

Answer: The line passes through P and is perpendicular to OP , so it is the tangent at P

Tip: Rearrange to $y = mx + c$ before reading off the gradient: the gradient of $4x - 3y = 50$ is $\frac{4}{3}$, not 4 or -3 . Finish with a sentence that says why it is a tangent.

Marks: 1 mark for showing P is on the line. 1 mark for the gradient of the line, $\frac{4}{3}$. 1 mark for the gradient of OP , $-\frac{3}{4}$. 1 mark for showing the product is -1 with a conclusion.

9 (6 marks)

The line $x + 2y = 10$ meets the circle $x^2 + y^2 = 25$ at the points A and B .

(a) Work out the coordinates of A and B .

You must show your working.

Rearrange the line to make x the subject: $x = 10 - 2y$

Substitute into the circle: $(10 - 2y)^2 + y^2 = 25$

Expand: $100 - 40y + 4y^2 + y^2 = 25$

$5y^2 - 40y + 75 = 0$

Divide by 5: $y^2 - 8y + 15 = 0$

Factorise: $(y - 3)(y - 5) = 0$, so $y = 3$ or $y = 5$

Find x using $x = 10 - 2y$:

$y = 3$ gives $x = 4$, and $y = 5$ gives $x = 0$

Answer: (4, 3) and (0, 5)

Tip: Making x the subject avoids fractions here. Find the other coordinate from the **line**, not the circle.

Marks: 1 mark for substituting. 1 mark for $y^2 - 8y + 15 = 0$ (or equivalent). 1 mark for both y values. 1 mark for both points.

- (b) O is the origin and M is the midpoint of AB .

Show that OM is perpendicular to AB .

$$M = \left(\frac{4+0}{2}, \frac{3+5}{2} \right) = (2, 4)$$

$$\text{Gradient of } OM = \frac{4}{2} = 2$$

$$\text{Gradient of } AB: x + 2y = 10 \text{ gives } y = -\frac{1}{2}x + 5, \text{ so the gradient is } -\frac{1}{2}$$

$$2 \times \left(-\frac{1}{2} \right) = -1, \text{ so } OM \text{ is perpendicular to } AB.$$

Answer: Gradients 2 and $-\frac{1}{2}$ multiply to -1 , so OM is perpendicular to AB

Tip: Two lines are perpendicular when their gradients multiply to -1 . (This is the circle theorem: the line from the centre to the midpoint of a chord is perpendicular to the chord.)

Marks: 1 mark for $M(2, 4)$ and the gradient of OM . 1 mark for the gradient of AB and showing the product is -1 .
