

Direct and Inverse Proportion - Forming Equations

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

y is directly proportional to $\frac{1}{x^2}$

Which statement is correct?

Tick one box.

Directly proportional to $\frac{1}{x^2}$ means $y = k \times \frac{1}{x^2}$, where k is a constant.

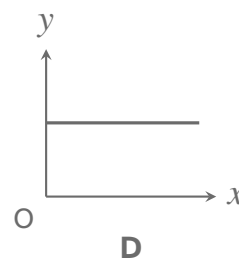
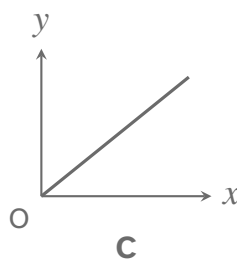
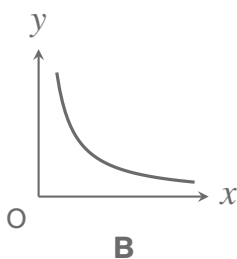
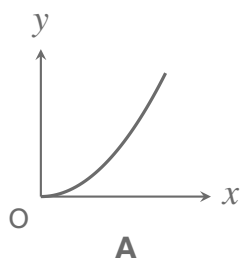
$$k \times \frac{1}{x^2} = \frac{k}{x^2}$$

So y is inversely proportional to x^2 .

Answer: y is inversely proportional to x^2

2 (1 mark)

Here are four sketch graphs.



In one of these graphs, y is inversely proportional to x .
Circle the letter of this graph.

Options: A, B, C, D

Inverse proportion: $y = \frac{k}{x}$

As x gets bigger, y gets smaller, but it never reaches 0.

So the graph is a curve that gets closer and closer to both axes.

A is a curve going up (like $y = kx^2$).

C is a straight line through the origin (direct proportion).

D is a horizontal line (y does not change).

Only B is a curve that comes down towards the x -axis.

Answer: B

Tip: An inverse proportion graph is a curve that never touches the axes. It is not a straight line sloping downwards.

3 (1 mark)

h is directly proportional to w^2

The value of h is multiplied by 25

By what number has w been multiplied? Circle your answer.

Options: $\times 5$, $\times 25$, $\times 50$, $\times 625$

$$h = kw^2$$

h is multiplied by 25, so w^2 must be multiplied by 25.

$25 = 5^2$, so w is multiplied by 5.

Answer: $\times 5$

Tip: Work backwards: h follows w^2 , so take the square root of 25. Do not square it.

4 (1 mark)

T is inversely proportional to n^2

The value of n is halved.

Carys says, "The value of T will also be halved."

Is she correct? Tick a box.

Give a reason for your answer.

$$T = \frac{k}{n^2}$$

If n is halved, then n^2 is multiplied by $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

n^2 is on the bottom of the fraction, so T is multiplied by 4.

Carys is not correct.

Answer: No. T is multiplied by 4 (it does not halve)

Tip: Inverse proportion works the opposite way: when n gets smaller, T gets bigger.

5 (2 marks)

p is inversely proportional to q .

Complete the table.

q	2	5	
p	30		4

Inverse proportion: $p = \frac{k}{q}$, so $k = q \times p$

$$k = 2 \times 30 = 60$$

$$\text{When } q = 5: p = 60 \div 5 = 12$$

$$\text{When } p = 4: q = 60 \div 4 = 15$$

Answer: 12 and 15

Tip: Do not use $p = kq$ (that is direct proportion). For inverse proportion, $q \times p$ is always the same number.

Marks: 1 mark for 12. 1 mark for 15.

6 (4 marks)

y is directly proportional to x^2

$$\text{When } x = 4, y = 48$$

(a) Find a formula for y in terms of x .

y is directly proportional to x^2 : $y = kx^2$

$$\text{Substitute } x = 4 \text{ and } y = 48: 48 = k \times 4^2$$

$$48 = 16k$$

$$k = 3$$

$$\text{So } y = 3x^2$$

Answer: $y = 3x^2$

Tip: Finish by writing the whole formula, $y = 3x^2$. Stopping at $k = 3$ loses a mark.

Marks: 1 mark for $y = kx^2$. 1 mark for $k = 3$. 1 mark for $y = 3x^2$.

(b) Work out y when $x = 5$

$$y = 3 \times 5^2 = 3 \times 25 = 75$$

Answer: 75

7 (5 marks)

F is inversely proportional to the square of d .

When $d = 3$, $F = 20$

(a) Find an equation for F in terms of d .

F is inversely proportional to the square of d : $F = \frac{k}{d^2}$

Substitute $d = 3$ and $F = 20$: $20 = \frac{k}{9}$

$$k = 20 \times 9 = 180$$

$$\text{So } F = \frac{180}{d^2}$$

Answer: $F = \frac{180}{d^2}$

Tip: "Inversely proportional to the square of d " means d^2 goes on the **bottom**: $F = \frac{k}{d^2}$, not $F = kd^2$.

Marks: 1 mark for $F = \frac{k}{d^2}$. 1 mark for $k = 180$. 1 mark for $F = \frac{180}{d^2}$.

(b) Work out the positive value of d when $F = 5$

$$5 = \frac{180}{d^2}$$

$$d^2 = 180 \div 5 = 36$$

$$d = \sqrt{36} = 6 \text{ (} d \text{ is positive)}$$

Answer: 6

Marks: 1 mark for $d^2 = 36$. 1 mark for 6.

8 (3 marks)

p is proportional to $\sqrt{q}$
Complete the table.

q	4	25	
p	6		21

p is proportional to $\sqrt{q}$: $p = k\sqrt{q}$

Substitute $q = 4$ and $p = 6$: $6 = k \times \sqrt{4} = 2k$, so $k = 3$

So $p = 3\sqrt{q}$

When $q = 25$: $p = 3 \times \sqrt{25} = 3 \times 5 = 15$

When $p = 21$: $21 = 3\sqrt{q}$, so $\sqrt{q} = 7$

Square both sides: $q = 7^2 = 49$

Answer: 15 and 49

Marks: 1 mark for $k = 3$ (or $p = 3\sqrt{q}$). 1 mark for 15. 1 mark for 49.

9 (4 marks)

The power, P kilowatts, produced by a small wind turbine is directly proportional to the cube of the wind speed, v metres per second.

When $v = 4$, $P = 32$

Work out the wind speed when the turbine produces 108 kilowatts.

P is directly proportional to v^3 : $P = kv^3$

When $v = 4$, $P = 32$: $32 = k \times 4^3 = 64k$, so $k = \frac{1}{2}$

So $P = \frac{1}{2}v^3$

When $P = 108$: $108 = \frac{1}{2}v^3$

$v^3 = 216$

$$v = \sqrt[3]{216} = 6$$

Answer: 6 metres per second

Tip: “The cube of the wind speed” means v^3 . At the end, take the **cube root** of 216, not the square root.

Marks: 1 mark for $P = kv^3$. 1 mark for $k = \frac{1}{2}$. 1 mark for $v^3 = 216$. 1 mark for 6.

10 (5 marks)

r is positive.

V is directly proportional to r^3

A is directly proportional to r^2

When $r = 2$, $V = 40$ and $A = 24$

Work out the value of V when $A = 54$

V is directly proportional to r^3 : $V = kr^3$

$$40 = k \times 2^3 = 8k, \text{ so } k = 5 \text{ and } V = 5r^3$$

A is directly proportional to r^2 : $A = cr^2$

$$24 = c \times 2^2 = 4c, \text{ so } c = 6 \text{ and } A = 6r^2$$

$$\text{When } A = 54: 54 = 6r^2, \text{ so } r^2 = 9$$

$$r = 3 \text{ (} r \text{ is positive)}$$

$$V = 5 \times 3^3 = 5 \times 27 = 135$$

Answer: 135

Tip: Use a different letter for each constant (such as k and c). Use the A formula to find r first, then put r into the V formula.

Marks: 1 mark for $V = kr^3$ with $k = 5$. 1 mark for $A = cr^2$ with $c = 6$. 1 mark for $r^2 = 9$. 1 mark for $r = 3$. 1 mark for 135.
