

Cumulative Frequency

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (8 marks)

The table shows the times taken by 80 people to complete a puzzle.

Time (t minutes)	Frequency
$0 < t \leq 10$	6
$10 < t \leq 20$	14
$20 < t \leq 30$	25
$30 < t \leq 40$	20
$40 < t \leq 50$	10
$50 < t \leq 60$	5

(a) Complete the cumulative frequency table.

Keep a running total of the frequencies:

6, $6 + 14 = 20$, $20 + 25 = 45$, $45 + 20 = 65$, $65 + 10 = 75$, $75 + 5 = 80$

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	6
$0 < t \leq 20$	20
$0 < t \leq 30$	45
$0 < t \leq 40$	65
$0 < t \leq 50$	75
$0 < t \leq 60$	80

The last value should equal the total number of people, 80 ✓

Answer: 6, 20, 45, 65, 75, 80

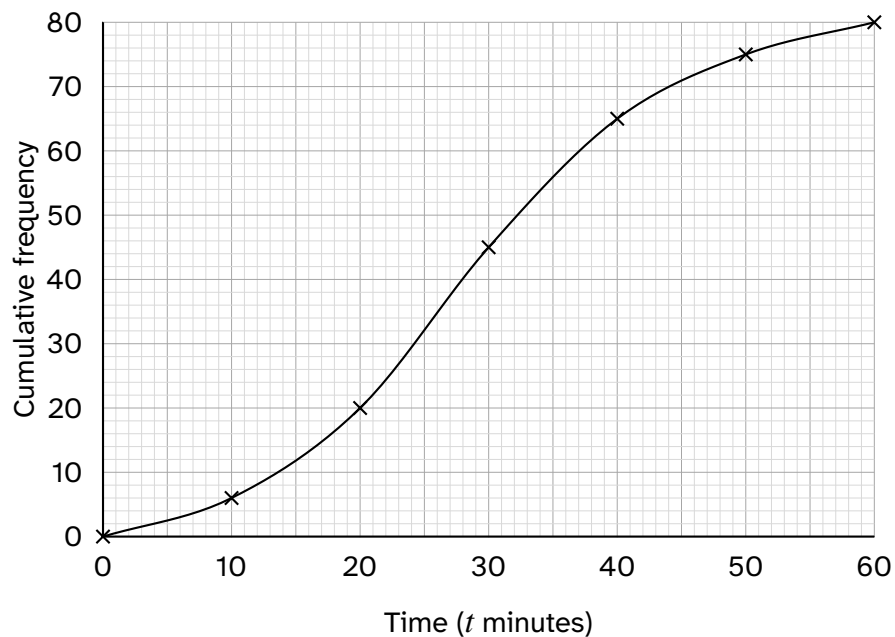
- (b) Draw a cumulative frequency graph on the grid.

Plot each cumulative frequency at the **end** of its class (the upper class boundary):

(10, 6), (20, 20), (30, 45), (40, 65), (50, 75), (60, 80)

The first class starts at 0, so also plot (0, 0).

Join the points with a smooth curve.



Answer: See graph

Tip: Do not plot at the midpoints (5, 15, ...). This is the most common mistake: the cumulative frequency is only complete at the end of each class.

Marks: 1 mark for plotting the points at the upper class boundaries. 1 mark for a smooth curve (or straight lines) through them.

- (c) Use your graph to estimate the median time.

The median is halfway through the data: half of 80 is 40.

Read across from 40 on the cumulative frequency axis to the curve, then down.

This gives about 28 minutes.

Answer: 28 (accept 27 to 29)

- (d) Use your graph to estimate the interquartile range.

Lower quartile: $\frac{1}{4}$ of 80 is 20. Read across from 20: LQ = 20 minutes.

Upper quartile: $\frac{3}{4}$ of 80 is 60. Read across from 60: UQ $\approx$ 37 minutes.

Interquartile range = $UQ - LQ = 37 - 20 = 17$ minutes

Answer: 17 (accept 16 to 19)

Tip: The interquartile range is $UQ - LQ$, not the range (largest – smallest).

Marks: 1 mark for reading the quartiles at 20 and 60. 1 mark for an answer from 16 to 19.

- (e) Estimate how many people took more than 45 minutes.

Read up from 45 minutes to the curve, then across: about 71.

So about 71 people took 45 minutes or less.

More than 45 minutes: $80 - 71 = 9$ people

Answer: 9 (accept 8 to 12)

Tip: The graph shows how many took 45 minutes **or less**. For “more than”, subtract your reading from the total.

Marks: 1 mark for reading the curve at 45 minutes. 1 mark for subtracting from 80.

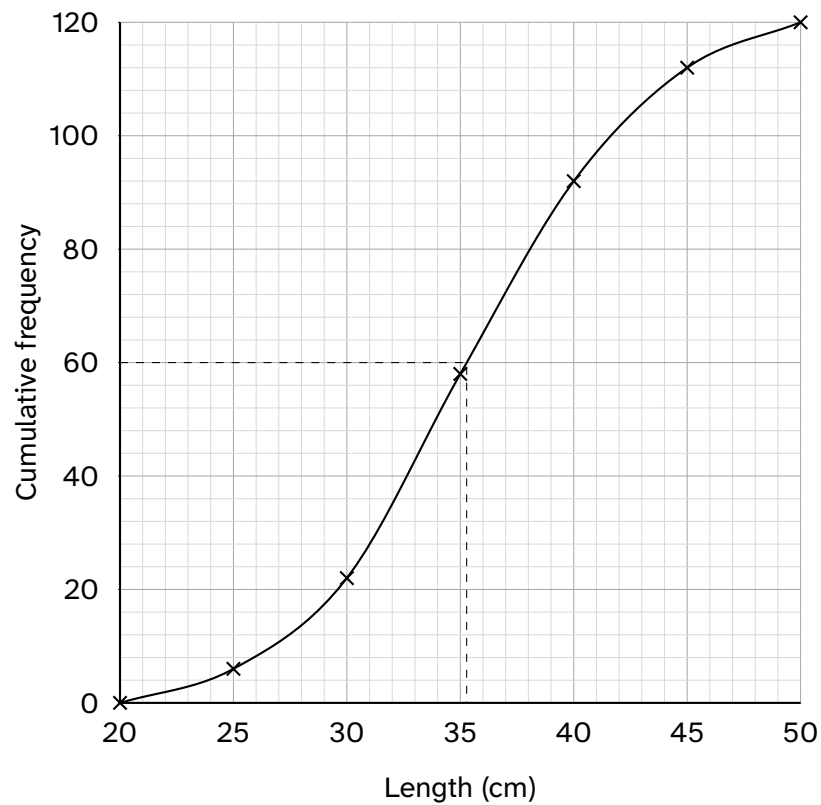
2 (6 marks)

The cumulative frequency graph shows information about the lengths of 120 fish caught in a lake.

- (a) Estimate the median length.

Half of 120 is 60.

Read across from 60 to the curve, then down.

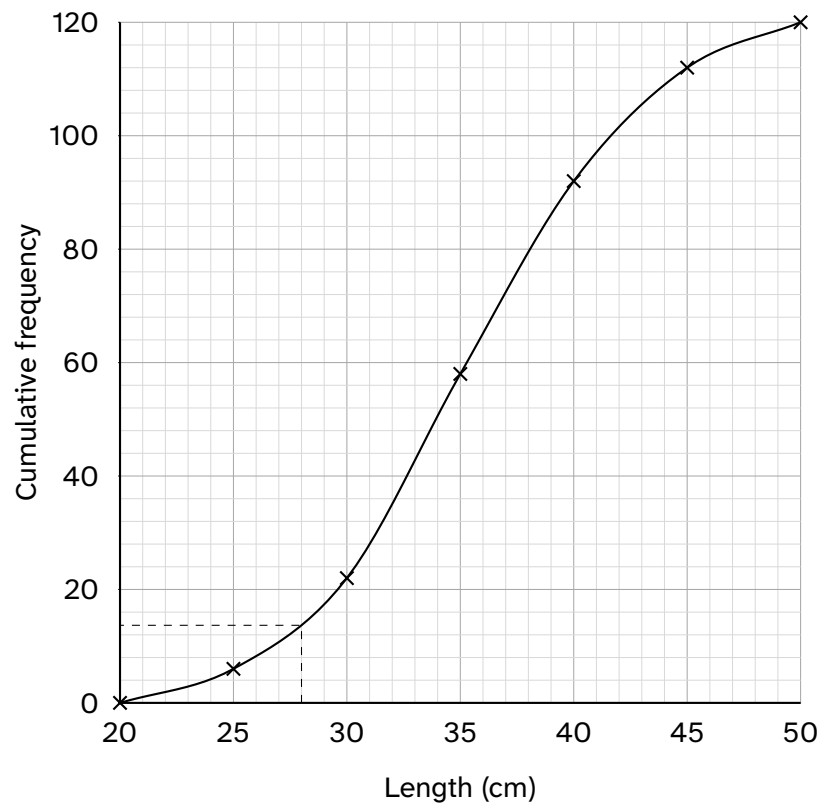


This gives about 35 cm.

Answer: 35 (accept 34.5 to 36)

- (b) Fish shorter than 28 cm must be put back into the lake.
All the other fish can be kept.
Estimate the percentage of these fish that can be kept.

Read up from 28 cm to the curve, then across: about 14 fish are shorter than 28 cm.



Fish that can be kept: $120 - 14 = 106$

Percentage: $\frac{106}{120} \times 100 = 88.3\%$ (1 dp)

Answer: 88.3% (accept 86.7% to 90%)

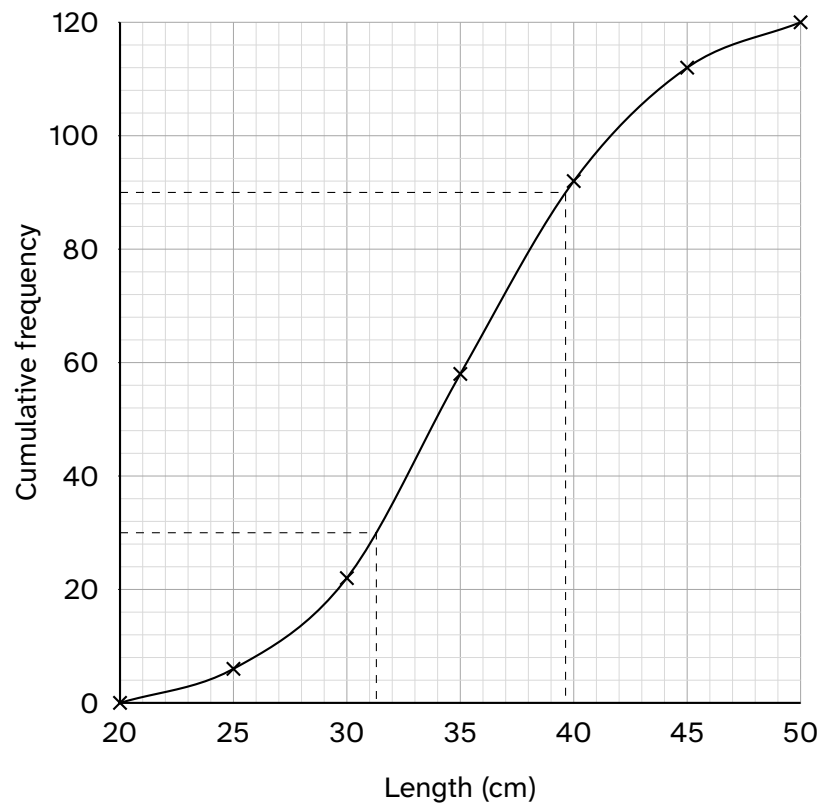
Tip: The graph gives the number of fish **up to** 28 cm. Subtract from 120 to find the number that can be kept, then change it to a percentage.

Marks: 1 mark for reading the graph at 28 cm (12 to 16). 1 mark for $120 -$ your reading. 1 mark for the correct percentage from your reading (86.7% to 90%).

(c) Estimate the interquartile range.

Lower quartile: $\frac{1}{4}$ of 120 is 30. Read across from 30: about 31.3 cm.

Upper quartile: $\frac{3}{4}$ of 120 is 90. Read across from 90: about 39.6 cm.



$$\text{IQR} \approx 39.6 - 31.3 = 8.3 \text{ cm}$$

Answer: 8.3 (accept 7.5 to 9.5)

Tip: Use $\frac{1}{4}$ and $\frac{3}{4}$ of the total frequency (30 and 90) on the cumulative frequency axis, then subtract the two **lengths**.

Marks: 1 mark for reading across from 30 and 90. 1 mark for an answer from 7.5 to 9.5.

3 (2 marks)

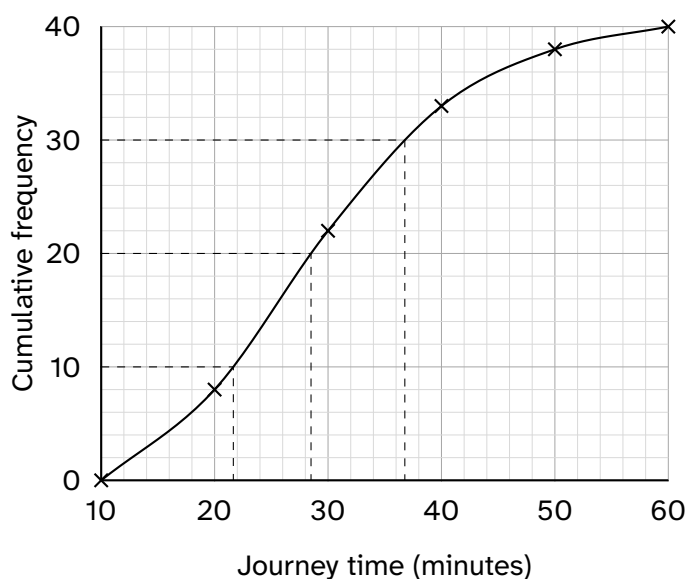
The cumulative frequency graph shows information about the journey times of 40 commuters.

Ravi uses the graph to work out the median and the interquartile range. Here is his working.

Median = $(10 + 60) \div 2 = 35$ minutes
Lower quartile: read across from 10, so LQ = 22 minutes
Upper quartile: read across from 30, so UQ = 37 minutes
Interquartile range = $30 - 10 = 20$ minutes

Make **two** criticisms of Ravi's working.

- 1.** The median is not halfway along the time axis. He should read across from half of 40, which is 20, on the cumulative frequency axis. This gives about 28.5 minutes.
- 2.** The interquartile range is the difference between the two quartile **times**: $37 - 22 = 15$ minutes. He subtracted the cumulative frequencies ($30 - 10$) instead.



Answer: Median should be read across from 20 (about 28.5 min), not the middle of the time axis; IQR should be $37 - 22 = 15$ minutes, not $30 - 10$

Tip: Be precise: say what Ravi did wrong and what he should have done. His quartile readings (from 10 and 30) are correct.

Marks: 1 mark for each correct criticism.

4 (7 marks)

Two brands of phone charging cable, P and Q , were tested.

100 cables of each brand were used every day until they stopped working.

The number of days each cable lasted was recorded.

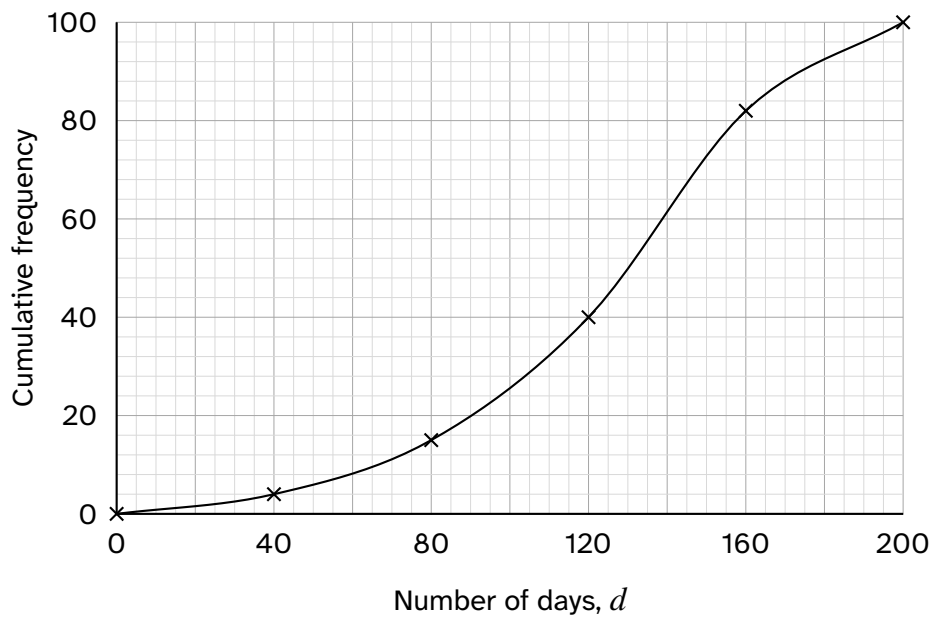
The frequency table shows the results for brand P .

Number of days, d	Frequency
$0 \leq d < 40$	4
$40 \leq d < 80$	11
$80 \leq d < 120$	25
$120 \leq d < 160$	42
$160 \leq d < 200$	18

- (a) On the grid, draw a cumulative frequency diagram to represent the data.

Cumulative frequencies (running totals): 4, 15, 40, 82, 100

Plot them at the upper class boundaries, starting at $(0, 0)$, and join with a smooth curve.



Answer: See graph

Marks: 1 mark for the cumulative frequencies. 1 mark for the points. 1 mark for the curve.

- (b) For brand Q ,
the median was 118 days
the interquartile range was 64 days.

Compare the number of days that brands P and Q lasted.

Make one statement about the average and one statement about the spread.

Use statistical measures to support your statements.

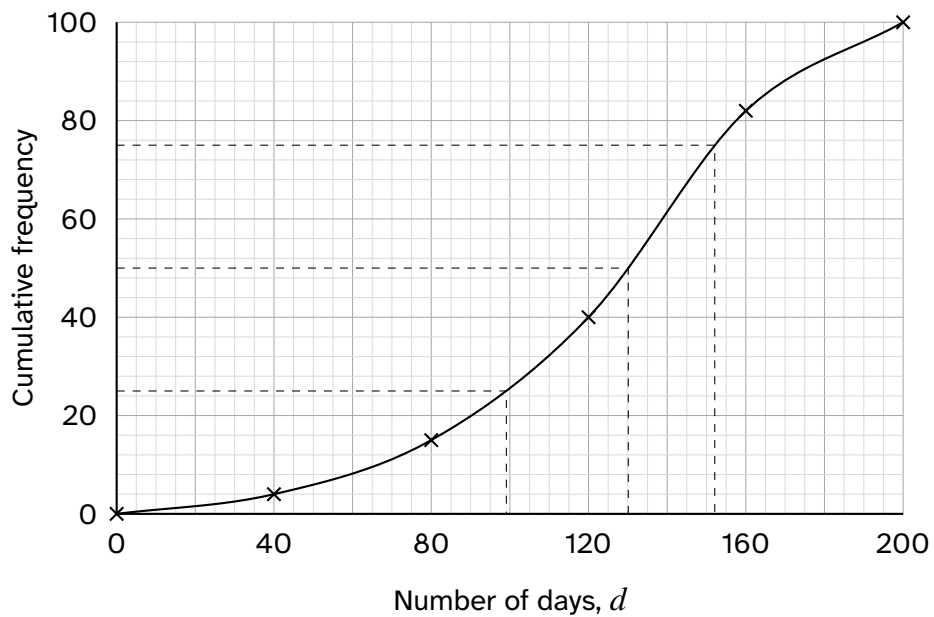
First find the median and IQR for brand P from your graph.

Median: read across from 50 (half of 100): about 130 days.

LQ: read across from 25: about 99 days.

UQ: read across from 75: about 152 days.

$IQR \approx 152 - 99 = 53$ days



Average: the median for brand P (about 130 days) is higher than for brand Q (118 days).

So brand P cables lasted longer on average.

Spread: the IQR for brand P (about 53 days) is smaller than for brand Q (64 days).

So the number of days brand P cables lasted was more consistent.

Answer: P median about 130, IQR about 53: P lasted longer on average and more consistently

Marks: 1 mark for the median of P (126 to 134). 1 mark for the IQR of P (48 to 58). 1 mark for comparing medians in context. 1 mark for comparing IQRs in context.

5 (5 marks)

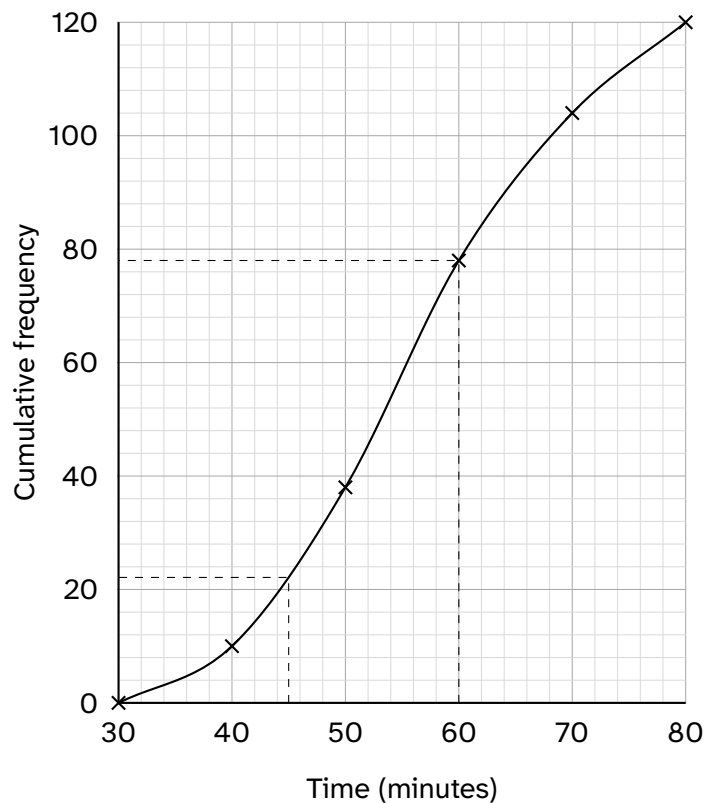
120 runners took part in a 10 km race.

The cumulative frequency graph shows information about their finishing times.

- (a) Estimate the percentage of the runners who finished in a time between 45 minutes and 60 minutes.

Read up from 45 minutes to the curve, then across: about 22 runners.

Read up from 60 minutes: 78 runners.



Between 45 and 60 minutes: $78 - 22 = 56$ runners

Percentage: $\frac{56}{120} \times 100 = 46.7\%$ (1 dp)

Answer: 46.7% (accept 45% to 48.5%)

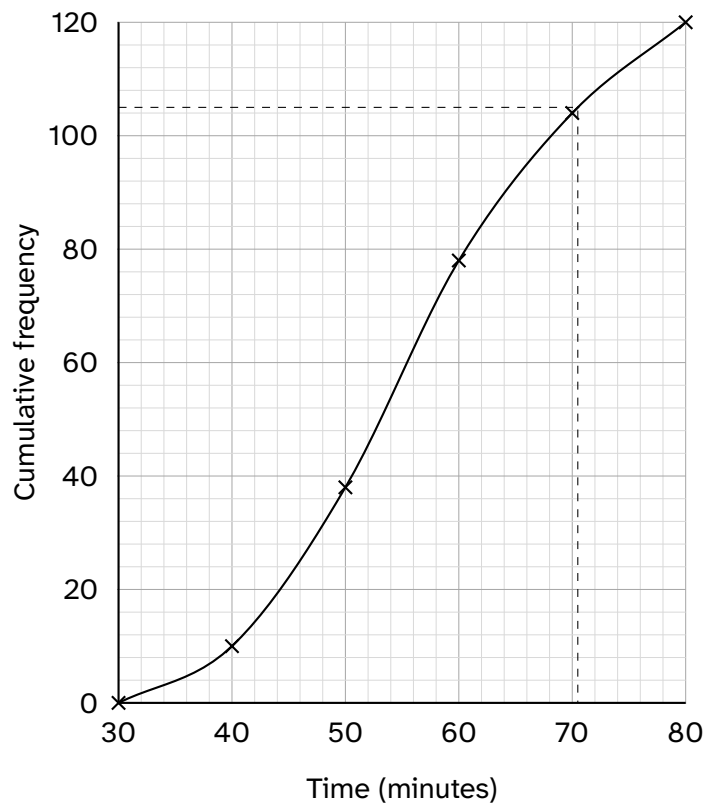
Tip: The graph shows running totals. Subtract the reading at 45 from the reading at 60 to find how many runners are in between.

Marks: 1 mark for reading the graph at 45 or 60 minutes. 1 mark for $78 -$ your reading at 45. 1 mark for the correct percentage (45% to 48.5%).

- (b) The slowest 15 runners each get a “Never give up” medal.
Estimate the shortest time taken by a runner who got this medal.

The slowest 15 runners are the last 15 of the 120.

$120 - 15 = 105$, so read across from 105 on the cumulative frequency axis.



This gives about 70.5 minutes.

Answer: 70.5 (accept 70 to 71.5)

Tip: The slowest runners are at the **top** of the graph, so read across from $120 - 15 = 105$, not from 15.

Marks: 1 mark for reading across from 105. 1 mark for an answer from 70 to 71.5.