

Cubic and Reciprocal Graphs

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

(a) Complete the table of values for $y = x^3$

x	-2	-1	0	1	2
y	-8	-1	0	1	8

x^3 means $x \times x \times x$.

$$x = -2: (-2) \times (-2) \times (-2) = 4 \times (-2) = -8$$

$$x = 0: 0 \times 0 \times 0 = 0$$

$$x = 2: 2 \times 2 \times 2 = 8$$

Answer: -8, 0, 8

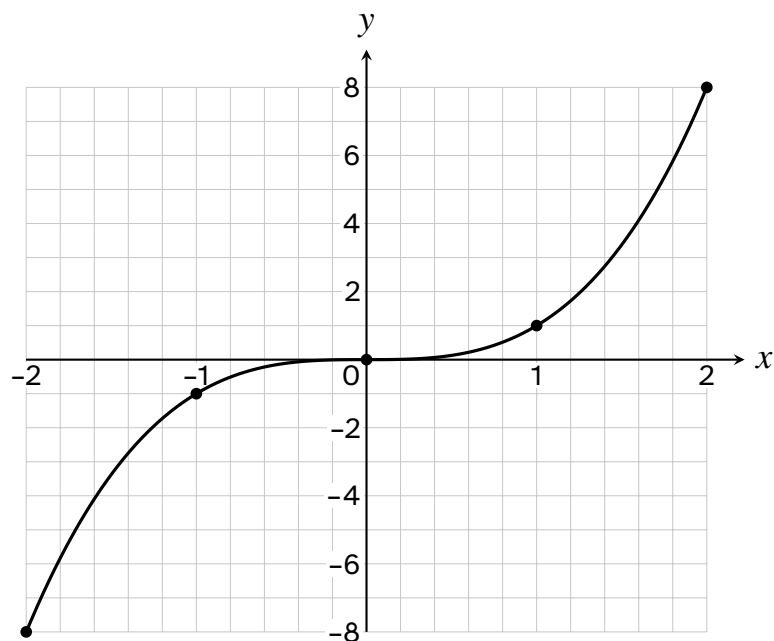
Tip: A negative number cubed is negative: $(-2)^3 = -8$, not +8 and not -6.

Marks: 2 marks for all three correct. 1 mark for two correct.

(b) Draw the graph of $y = x^3$ for values of x from -2 to 2

Plot the five points: $(-2, -8)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 8)$.

Join them with one smooth curve.

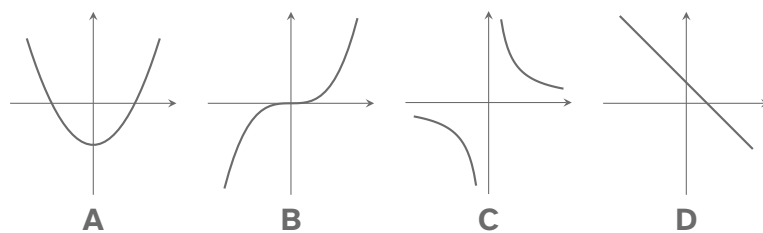


Tip: Join the points with a smooth curve, not with straight lines. The curve should flatten out as it passes through (0,0).

Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

2 (3 marks)

Here are four sketch graphs.



- (a) Write down the letter of the graph of $y = x^3$

$y = x^3$ has the cubic "S" shape.

It goes from bottom-left to top-right and flattens out at the origin.

That is graph B.

Answer: B

- (b) Write down the letter of the graph of $y = \frac{1}{x}$

$y = \frac{1}{x}$ has two separate parts that never touch the axes.

That is graph C.

Answer: C

- (c) Write down the letter of the graph of $y = x^2 - 1$

$y = x^2 - 1$ is a U-shaped curve (a parabola).

It crosses the y-axis at -1 .

That is graph A.

Answer: A

3 (4 marks)

- (a) Complete the table of values for $y = \frac{4}{x}$

x	0.5	1	2	4	8
y	8	4	2	1	0.5

$\frac{4}{x}$ means $4 \div x$.

$$x = 0.5: 4 \div 0.5 = 8$$

$$x = 2: 4 \div 2 = 2$$

$$x = 4: 4 \div 4 = 1$$

Answer: 8, 2, 1

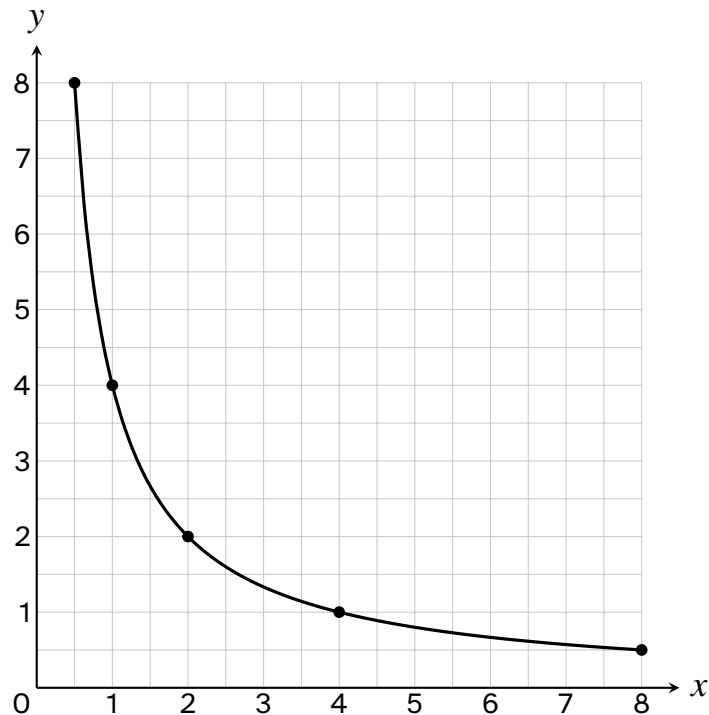
Tip: Dividing by 0.5 doubles the number: $4 \div 0.5 = 8$, not 2.

Marks: 2 marks for all three correct. 1 mark for two correct.

- (b) Draw the graph of $y = \frac{4}{x}$ for values of x from 0.5 to 8

Plot the five points: (0.5, 8), (1, 4), (2, 2), (4, 1), (8, 0.5).

Join them with one smooth curve.



Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

4 (1 mark)

Explain why the graph of $y = \frac{1}{x}$ never crosses the y-axis.

Every point on the y-axis has $x = 0$.

When $x = 0$, $y = \frac{1}{0}$.

You cannot divide by 0, so there is no point on the graph with $x = 0$.

Answer: $x = 0$ is not possible because you cannot divide by 0.

5 (4 marks)

- (a) Complete the table of values for $y = x^3 - 3x$

x	-2	-1	0	1	2
y	-2	2	0	-2	2

$$x = -1: (-1)^3 - 3 \times (-1) = -1 + 3 = 2$$

$$x = 1: 1^3 - 3 \times 1 = 1 - 3 = -2$$

Answer: 2, -2

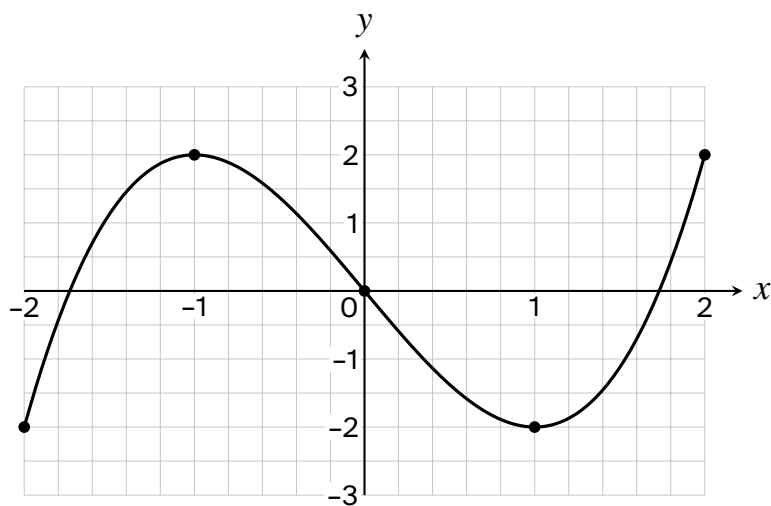
Marks: 2 marks for both correct. 1 mark for one correct.

- (b) Draw the graph of $y = x^3 - 3x$ for values of x from -2 to 2

Plot the five points: $(-2, -2)$, $(-1, 2)$, $(0, 0)$, $(1, -2)$, $(2, 2)$.

Join them with one smooth curve.

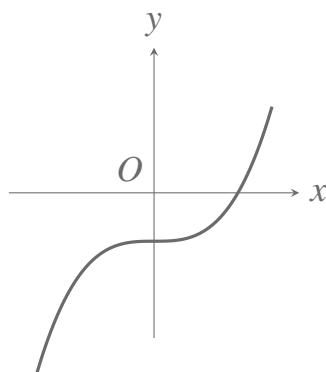
The curve turns at $x = -1$ and at $x = 1$.



Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

6 (1 mark)

Here is a sketch of the graph of $y = x^3 + c$ where c is a constant.



Tick **one** box.

When $x = 0$, $y = 0^3 + c = c$

So the graph crosses the y-axis at $(0, c)$.

On the sketch, the graph crosses the y-axis **below** O .

So c is negative.

Answer: $c < 0$

Tip: $y = x^3 + c$ is the graph of $y = x^3$ moved up (if c is positive) or down (if c is negative). This one has been moved down.

7 (1 mark)

Which point lies on the graph of $y = \frac{1}{x}$?

Circle your answer.

Options: $(0, 1)$, $(1, 0)$, $(-1, 1)$, $(2, 0.5)$

Substitute the x -coordinate into $y = \frac{1}{x}$ and check the y -coordinate.

$x = 0$: $1 \div 0$ has no value, so $(0, 1)$ is not on the graph.

$x = 1$: $y = 1 \div 1 = 1$, not 0.

$x = -1$: $y = 1 \div (-1) = -1$, not 1.

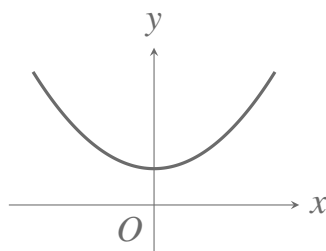
$$x = 2: y = 1 \div 2 = 0.5 \quad \checkmark$$

Answer: (2, 0.5)

Tip: The graph of $y = \frac{1}{x}$ never meets either axis, so a point with a 0 in it can never be on it.

8 (2 marks)

Here is Ffion's sketch of the graph of $y = x^3$



Give **two** reasons why her sketch is wrong.

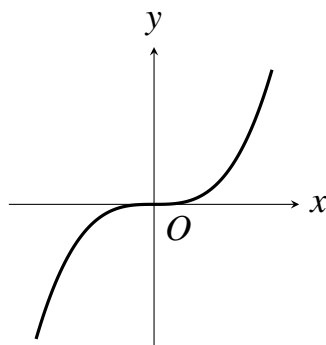
Reason 1: when $x = 0$, $y = 0^3 = 0$, so the graph must pass through the origin O .

Ffion's graph crosses the y -axis above O .

Reason 2: when x is negative, x^3 is negative (e.g. $(-1)^3 = -1$).

So the left-hand part should be below the x -axis, but Ffion's is above it.

Correct sketch:



Answer: It should pass through O ; for negative x the graph should be below the x -axis

Tip: Ffion has drawn a U-shaped (quadratic) curve. A cubic like $y = x^3$ goes from bottom-left to top-right. Be precise: say exactly which part is wrong.

Marks: 1 mark for each different correct reason.

9 (4 marks)

A fish tank holds 36 litres of water.

A pump fills the empty tank at a rate of r litres per minute.

The time, T minutes, it takes to fill the tank is given by $T = \frac{36}{r}$

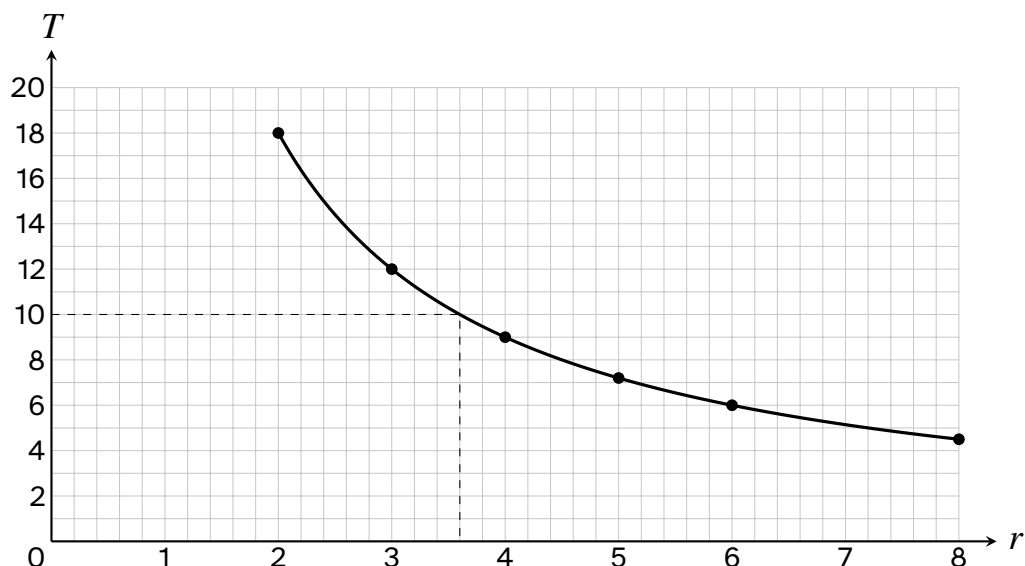
- (a) On the grid below, draw the graph of $T = \frac{36}{r}$ for values of r from 2 to 8

Make a table of values using $T = 36 \div r$:

$$r = 2: T = 18 \quad r = 3: T = 12 \quad r = 4: T = 9$$

$$r = 5: T = 7.2 \quad r = 6: T = 6 \quad r = 8: T = 4.5$$

Plot the points and join them with a smooth curve.



Tip: Use a smooth curve through all the points. Don't join them with straight lines and don't draw past $r = 8$.

Marks: 1 mark for at least 4 correct points plotted. 1 mark for a smooth curve from $r = 2$ to $r = 8$.

- (b) Nisha's pump fills the tank in 10 minutes.
Use your graph to estimate the rate of Nisha's pump.

T is the time, so use $T = 10$.

Read across from $T = 10$ to the curve, then down to the r -axis (the dashed line).

$r \approx 3.6$

Check: $36 \div 3.6 = 10$ ✓

Any answer from 3.4 to 3.8 is fine.

Answer: about 3.6 litres per minute (accept 3.4 to 3.8)

Tip: Read from the correct axis. 10 is a value of T (the time), so start on the T -axis and read the answer on the r -axis.

Marks: 1 mark for reading across from $T = 10$. 1 mark for an answer from 3.4 to 3.8.