

Congruent Triangles and Geometric Proof

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Two triangles have the same three angles.

Kit says,

‘The two triangles must be congruent.’

Is Kit correct? Tick a box.

Give a reason for your answer.

Congruent means exactly the same shape **and** the same size.

Equal angles only show that the triangles are the same shape (similar).

For example, two equilateral triangles with sides 2 cm and 5 cm have the same angles but are different sizes.

So Kit is not correct: AAA is not a condition for congruence.

Answer: No: equal angles only show the triangles are similar; they could be different sizes

Tip: The conditions for congruence are SSS, SAS, ASA and RHS. Each one needs at least one pair of equal sides.

2 (1 mark)

In triangle PQR and triangle XYZ

$$PQ = XY = 7 \text{ cm} \quad QR = YZ = 11 \text{ cm} \quad \text{angle } PQR = \text{angle } XYZ = 48^\circ$$

Circle the condition that shows these triangles are congruent.

Options: SSS, SAS, ASA, RHS

Two pairs of sides are equal: $PQ = XY$ and $QR = YZ$.

Angle PQR is at Q , between sides PQ and QR .

Angle XYZ is at Y , between sides XY and YZ .

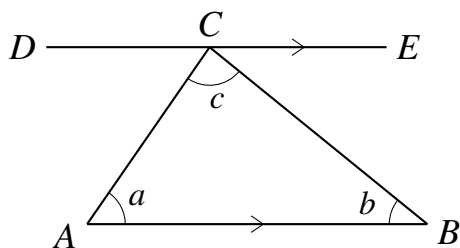
So the equal angles are the angles between the equal sides: this is SAS.

Answer: SAS

Tip: For SAS the angle must be the one **between** the two sides. Use the letters: the middle letter of angle PQR is Q , which is on both PQ and QR .

3 (3 marks)

ABC is a triangle.



Not drawn accurately

DCE is a straight line parallel to AB .

Prove that $a + b + c = 180^\circ$

DE is parallel to AB , so we can use alternate angles.

Angle $DCA = a$ (alternate angles are equal)

Angle $ECB = b$ (alternate angles are equal)

Angles DCA , c and ECB lie on the straight line DCE .

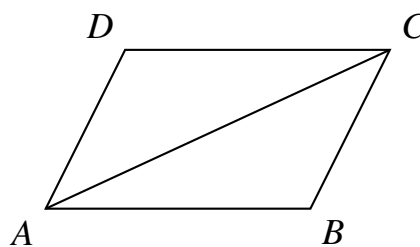
So $a + c + b = 180^\circ$ (angles on a straight line add up to 180°)

Tip: Write each reason in full, e.g. “alternate angles are equal”. Writing just “alternate” or “Z angles” may not get the mark.

Marks: 1 mark for one pair of alternate angles with a reason. 1 mark for both pairs. 1 mark for a complete proof with all the reasons.

4 (3 marks)

Here is a parallelogram.



Not drawn accurately

$ABCD$ is a parallelogram.

Prove that triangles ABC and CDA are congruent.

$AB = CD$ (opposite sides of a parallelogram are equal)

$BC = DA$ (opposite sides of a parallelogram are equal)

AC is a side of both triangles (a common side).

All three pairs of sides are equal.

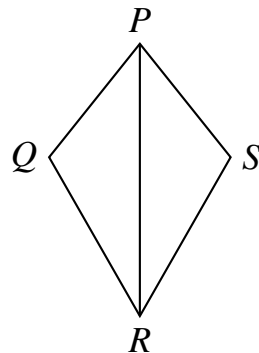
So triangles ABC and CDA are congruent (SSS).

Tip: Name sides with two letters, e.g. $AB = CD$. Writing “ $A = C$ ” does not say which sides you mean.

Marks: 1 mark for one pair of equal sides with a reason. 1 mark for all three pairs. 1 mark for concluding they are congruent by SSS.

5 (3 marks)

Here is a kite.



Not drawn accurately

$PQRS$ is a kite with $PQ = PS$ and $RQ = RS$.

Prove that angle $PQR =$ angle PSR .

Look at triangles PQR and PSR .

$PQ = PS$ (given)

$RQ = RS$ (given)

PR is a side of both triangles (common).

So triangles PQR and PSR are congruent (SSS).

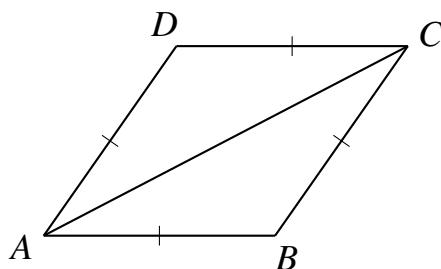
So angle $PQR =$ angle PSR (corresponding angles of congruent triangles are equal)

Tip: Prove the triangles are congruent first, then use that to say the angles are equal.

Marks: 1 mark for the equal pairs of sides. 1 mark for congruent by SSS. 1 mark for concluding the angles are equal.

6 (3 marks)

Here is a rhombus.



Not drawn accurately

$ABCD$ is a rhombus.

Prove that the diagonal AC bisects angle DAB .

Look at triangles ABC and ADC .

$AB = AD$ (all sides of a rhombus are equal)

$CB = CD$ (all sides of a rhombus are equal)

AC is a side of both triangles (common).

So triangles ABC and ADC are congruent (SSS).

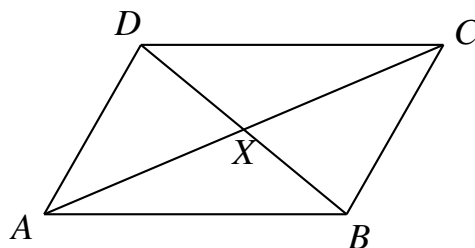
So angle $BAC = \text{angle } DAC$ (corresponding angles of congruent triangles)

“Bisects” means cuts into two equal parts, so AC bisects angle DAB .

Marks: 1 mark for equal sides with a reason. 1 mark for congruent by SSS. 1 mark for the equal angles, so AC bisects angle DAB .

7 (3 marks)

$ABCD$ is a parallelogram. The diagonals meet at X .



Not drawn accurately

For each statement, tick the correct box.

	True	May be true	Not true
Triangles AXB and CXD are congruent	✓		
$AC = BD$		✓	
Triangles ABX and ABC are congruent			✓
Angle $BAC = \text{angle } DCA$	✓		

Triangles AXB and CXD : $AB = CD$ (opposite sides of a parallelogram).

Two pairs of alternate angles are equal.

So they are congruent (ASA). **True**

$AC = BD$: the diagonals are only equal if the parallelogram is a rectangle. **May be true**

Triangles ABX and ABC : ABX is only part of triangle ABC , so it is smaller. **Not true**

Angles BAC and DCA : these are alternate angles, because AB is parallel to DC .

True

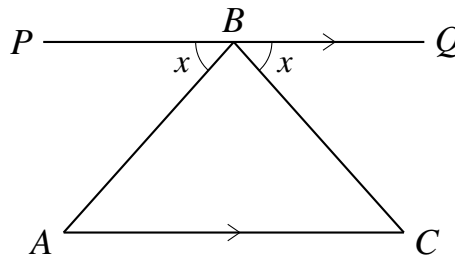
Answer: True, May be true, Not true, True

Tip: “May be true” means it is true for some parallelograms (such as a rectangle) but not for all of them.

Marks: 3 marks for all four correct. 2 marks for three correct. 1 mark for two correct.

8 (3 marks)

Here is a triangle and a straight line.



Not drawn accurately

ABC is a triangle.

PBQ is a straight line parallel to AC .

Angle $ABP = x$ and angle $CBQ = x$

Prove that $AB = BC$

PQ is parallel to AC , so we can use alternate angles.

Angle $BAC = \text{angle } ABP = x$ (alternate angles are equal)

Angle $BCA = \text{angle } CBQ = x$ (alternate angles are equal)

So angle $BAC = \text{angle } BCA$.

A triangle with two equal angles is isosceles.

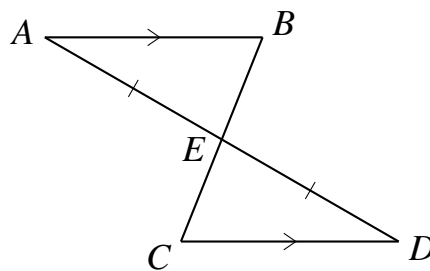
The equal sides are opposite the equal angles, so $AB = BC$.

Tip: Name each angle with three letters (angle BAC , not angle A) so it is clear which angle you mean.

Marks: 1 mark for one angle in triangle ABC equal to x with a reason. 1 mark for both. 1 mark for saying the triangle is isosceles, so $AB = BC$.

9 (4 marks)

Here is a diagram.



Not drawn accurately

AB is parallel to DC .

AD and BC are straight lines that cross at E .

E is the midpoint of AD .

Prove that triangle ABE is congruent to triangle DCE .

$AE = DE$ (E is the midpoint of AD)

Angle $BAE =$ angle CDE (alternate angles are equal, as AB is parallel to DC)

Angle $AEB =$ angle DEC (vertically opposite angles are equal)

So we have two angles and the side between them.

So triangles ABE and DCE are congruent (ASA).

Tip: Write the equal angles in full, e.g. angle $AEB =$ angle DEC , not “angle $E =$ angle E ”. Use congruence reasons (SSS, SAS, ASA, RHS), not circle theorems.

Marks: 1 mark for $AE = DE$. 1 mark for one pair of equal angles. 1 mark for the second pair of equal angles. 1 mark for all the reasons and ASA.
