

Conditional Probability

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The table shows how 100 students travel to school.

	Walk	Bus	Car	Total
Girls	18	22	10	50
Boys	25	15	10	50

A student is chosen at random.

- (a) Given that the student is a girl, work out the probability that she walks to school.

“Given that the student is a girl” means we only look at the 50 girls.

18 of the girls walk.

Answer: $\frac{18}{50}$

- (b) Given that the student travels by car, work out the probability that the student is a boy.

Only look at the students who travel by car: $10 + 10 = 20$

10 of them are boys.

Answer: $\frac{10}{20}$

2 (5 marks)

A bag contains 7 red and 3 blue counters.

Two counters are taken at random without replacement.

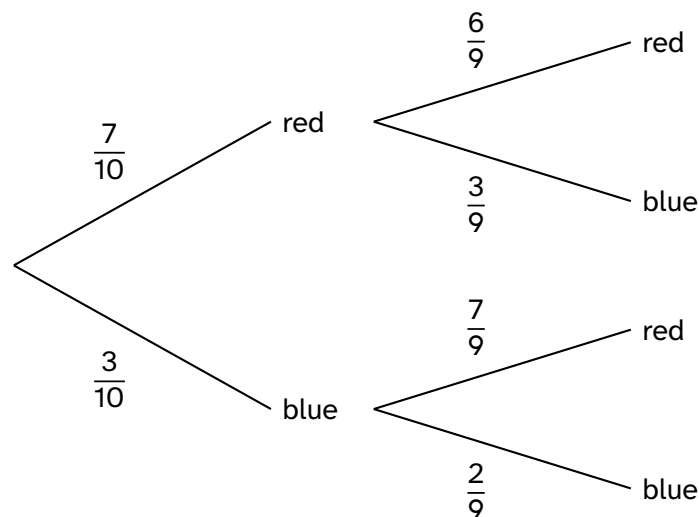
(a) Complete the tree diagram.

First counter: red $\frac{7}{10}$, blue $\frac{3}{10}$

The counter is not put back, so there are 9 counters left.

After a red: 6 red and 3 blue left, so red $\frac{6}{9}$, blue $\frac{3}{9}$

After a blue: 7 red and 2 blue left, so red $\frac{7}{9}$, blue $\frac{2}{9}$



Answer: See solution

Tip: Without replacement, the second pick is out of 9 (one fewer), not 8 or 10. The numbers on the second branches depend on what was taken first.

Marks: 1 mark for $\frac{3}{10}$. 1 mark for both second pairs correct.

(b) Work out the probability that both counters are the same colour.

Same colour: red-red or blue-blue.

Red, red: $\frac{7}{10} \times \frac{6}{9} = \frac{42}{90}$

Blue, blue: $\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

Add: $\frac{42}{90} + \frac{6}{90} = \frac{48}{90}$

Answer: $\frac{48}{90}$

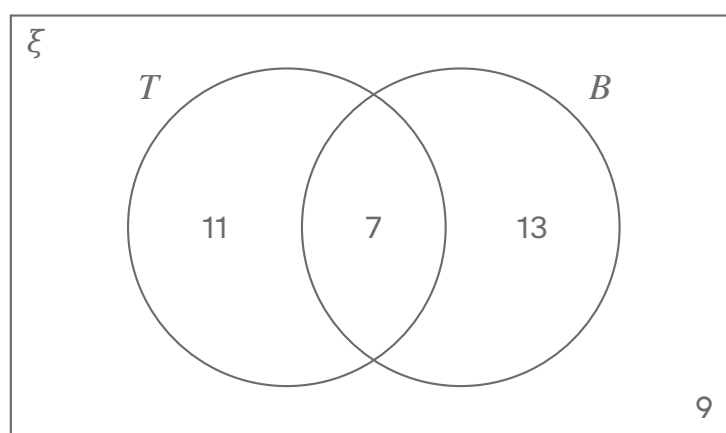
Marks: 1 mark for one correct product. 1 mark for adding both products. 1 mark for $\frac{48}{90}$ or an equivalent.

3 (4 marks)

The Venn diagram shows information about the 40 members of a sports club.

T = members who play tennis

B = members who play badminton



- (a) A member who plays badminton is chosen at random.
Work out the probability that this member also plays tennis.

We know the member plays badminton, so only look inside circle B .

Members who play badminton: $7 + 13 = 20$

7 of them also play tennis.

Answer: $\frac{7}{20}$

Tip: The bottom of the fraction is the number who play badminton (20), not all 40 members.

- (b) A member who plays exactly **one** of the two sports is chosen at random.
Work out the probability that this member plays tennis.

Exactly one sport means tennis only or badminton only: $11 + 13 = 24$

11 of these play tennis (only).

Answer: $\frac{11}{24}$

(c) Sam says,

“The probability that a badminton player also plays tennis is the same as the probability that a tennis player also plays badminton.”

Is Sam correct?

You must show your working.

A badminton player also plays tennis: $\frac{7}{20}$ (from part (a))

Tennis players: $11 + 7 = 18$

A tennis player also plays badminton: $\frac{7}{18}$

$\frac{7}{20} \neq \frac{7}{18}$ (the bottoms are different), so Sam is not correct.

Answer: No, $\frac{7}{20} \neq \frac{7}{18}$

Tip: The overlap (7) is the same, but you divide by a different group each time: 20 badminton players or 18 tennis players.

Marks: 1 mark for $\frac{7}{18}$. 1 mark for comparing it with $\frac{7}{20}$ and saying Sam is wrong.

4 (5 marks)

At a bakery, 4% of the cakes made are underweight.

A machine checks every cake.

If a cake is underweight, the probability that the machine rejects it is 0.9

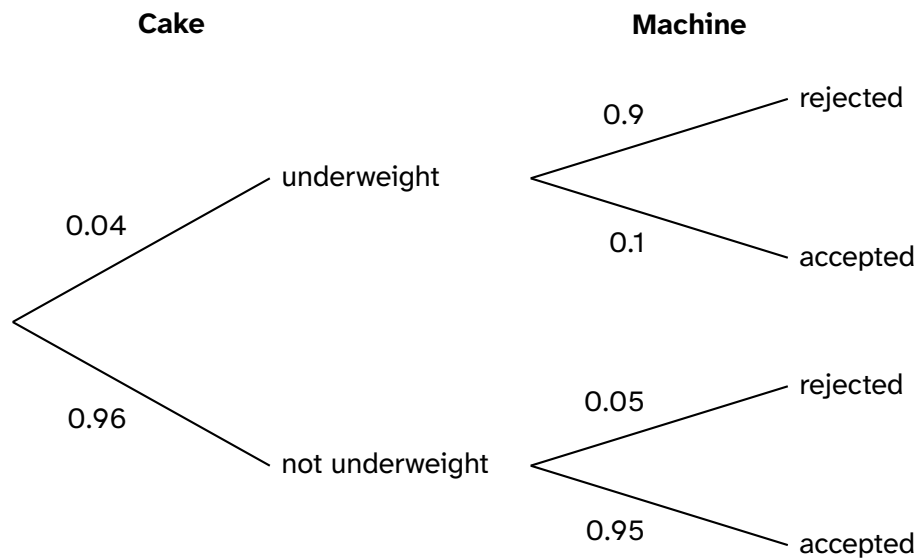
If a cake is not underweight, the probability that the machine rejects it is 0.05

(a) Complete the tree diagram.

Not underweight: $1 - 0.04 = 0.96$

Underweight cakes: rejected 0.9, accepted $1 - 0.9 = 0.1$

Cakes that are not underweight: rejected 0.05, accepted $1 - 0.05 = 0.95$



Answer: 0.96; 0.1; 0.05, 0.95

Tip: The second-stage probabilities are different on each branch. The 0.05 goes on the “not underweight” branch.

Marks: 1 mark for 0.96 and 0.1. 1 mark for 0.05 and 0.95.

- (b) A cake is chosen at random.
Work out the probability that the machine rejects it.

A cake can be rejected in two ways:

Underweight and rejected: $0.04 \times 0.9 = 0.036$

Not underweight and rejected: $0.96 \times 0.05 = 0.048$

Add: $0.036 + 0.048 = 0.084$

Answer: 0.084

Tip: Some good cakes are rejected too. Include the “not underweight, rejected” path as well as the “underweight, rejected” path.

Marks: 1 mark for 0.04×0.9 or 0.96×0.05 . 1 mark for adding the two products. 1 mark for 0.084.

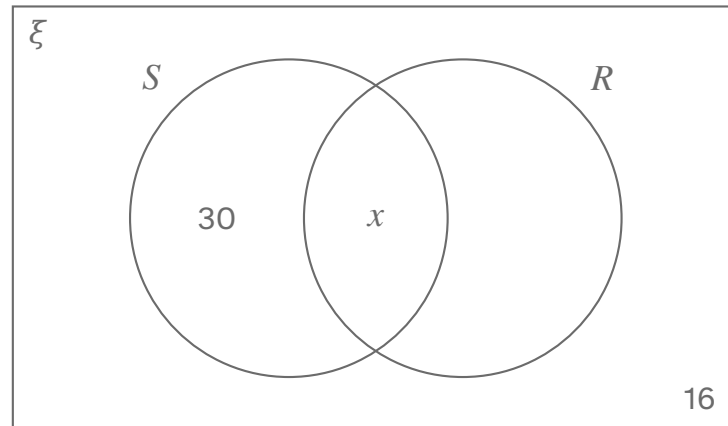
5 (4 marks)

The Venn diagram shows information about the 90 members of a gym.

ξ = 90 members

S = members who swim

R = members who run



A member who swims is chosen at random.

The probability that this member also runs is $\frac{1}{4}$

A member who runs is chosen at random.

Work out the probability that this member also swims.

Swimmers: $30 + x$, and x of them also run.

$$\text{So } \frac{x}{30 + x} = \frac{1}{4}$$

$$4x = 30 + x$$

$$3x = 30, \text{ so } x = 10$$

$$\text{Run only: } 90 - 30 - 10 - 16 = 34$$

$$\text{Runners: } 10 + 34 = 44$$

$$P(\text{swims} \mid \text{runs}) = \frac{10}{44}$$

Answer: $\frac{10}{44} \left(= \frac{5}{22} \right)$

Tip: $\frac{1}{4}$ is a fraction of the **swimmers** only (the whole S circle, $30 + x$), not of all 90 members.

Marks: 1 mark for $\frac{x}{30+x} = \frac{1}{4}$. 1 mark for $x = 10$. 1 mark for 34 (or 44 runners). 1 mark for $\frac{10}{44}$ or an equivalent.

6 (4 marks)

A choir has 12 singers.

n of the singers are sopranos.

There are fewer sopranos than other singers.

Two of the singers are chosen at random to sing a duet.

The probability that exactly one of the two singers is a soprano is $\frac{16}{33}$

Work out the value of n .

You must show your working.

Exactly one soprano: soprano then not, **or** not then soprano.

Soprano then not: $\frac{n}{12} \times \frac{12-n}{11}$

Not then soprano: $\frac{12-n}{12} \times \frac{n}{11}$

Total: $\frac{2n(12-n)}{132} = \frac{16}{33}$

Multiply both sides by 132: $2n(12 - n) = 64$

$n(12 - n) = 32$, so $12n - n^2 = 32$

$n^2 - 12n + 32 = 0$

Factorise: $(n - 4)(n - 8) = 0$

$n = 4$ or $n = 8$

There are fewer sopranos than other singers, so $n < 6$ and $n = 4$.

Answer: 4

Tip: “Exactly one” can happen in two orders, so double the product. Both 4 and 8 solve the equation; use the extra information to choose.

Marks: 1 mark for $\frac{n}{12} \times \frac{12-n}{11}$ (either order). 1 mark for doubling it and setting equal to $\frac{16}{33}$. 1 mark for $n^2 - 12n + 32 = 0$. 1 mark for $n = 4$ with 8 rejected.

7 (3 marks)

A tin contains 12 biscuits.

5 are chocolate, 4 are plain and 3 are ginger.

Dev takes two biscuits at random from the tin without replacement.

Given that the two biscuits are the same type, work out the probability that they are both chocolate.

Two biscuits are taken without replacement, out of 12 then 11.

$$\text{Both chocolate: } \frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$$

$$\text{Both plain: } \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$$

$$\text{Both ginger: } \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$$

$$\text{Same type: } \frac{20+12+6}{132} = \frac{38}{132}$$

$$\text{We know they are the same type, so divide: } \frac{20}{132} \div \frac{38}{132} = \frac{20}{38}$$

Answer: $\frac{20}{38} \left(= \frac{10}{19} \right)$

Tip: “Given that they are the same type” means the 38 same-type outcomes are the new total. $\frac{20}{132}$ is the answer to a different question.

Marks: 1 mark for $\frac{5}{12} \times \frac{4}{11}$. 1 mark for $\frac{38}{132}$ (same type). 1 mark for $\frac{20}{38}$ or an equivalent.
