

Compound Measures

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the speed that is the same as 15 metres per second.

Options: 4.2 km/h, 25 km/h, 54 km/h, 900 km/h

In 1 second you travel 15 m.

In 1 hour (3600 seconds): $15 \times 3600 = 54\,000$ m

$54\,000$ m = 54 km

So 15 m/s = 54 km/h

Answer: 54 km/h

Tip: Multiply by 3600 to change per second to per hour, then divide by 1000 to change m to km. Overall: $\times 3.6$

2 (2 marks)

A car travels 150 km in 1 hour 40 minutes.

Work out its average speed in km/h.

40 minutes = $\frac{40}{60} = \frac{2}{3}$ hour

So the time is $1\frac{2}{3} = \frac{5}{3}$ hours.

Speed = distance $\div$ time

$$150 \div \frac{5}{3} = 90 \text{ km/h}$$

Answer: 90 km/h

Tip: 1 hour 40 minutes is **not** 1.4 hours. 40 minutes is $\frac{40}{60} = 0.666 \dots$ of an hour.

Marks: 1 mark for $150 \div 1\frac{2}{3}$ (or $150 \div 100 \times 60$). 1 mark for 90.

3 (2 marks)

A washing machine stands on four feet.

Each foot touches the floor over an area of 12 cm^2

The washing machine exerts a force of 900 N on the floor.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the floor.

$$\text{Total area of the four feet: } 4 \times 12 = 48 \text{ cm}^2$$

$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

$$900 \div 48 = 18.75 \text{ N/cm}^2$$

Answer: 18.75 N/cm^2

Tip: The force is spread over all four feet, so use the total area $4 \times 12 = 48 \text{ cm}^2$, not 12 cm^2 .

Marks: 1 mark for $900 \div 48$. 1 mark for 18.75

4 (3 marks)

A solid cuboid measures 5 cm by 4 cm by 2 cm.

The cuboid has mass 312 g.

A solid cube of side 3 cm is made from the same metal.

Work out the mass of the cube.

$$\text{Volume of the cuboid: } 5 \times 4 \times 2 = 40 \text{ cm}^3$$

$$\text{Density of the metal: } 312 \div 40 = 7.8 \text{ g/cm}^3$$

$$\text{Volume of the cube: } 3 \times 3 \times 3 = 27 \text{ cm}^3$$

Mass of the cube = density $\times$ volume

$$7.8 \times 27 = 210.6 \text{ g}$$

Answer: 210.6 g

Tip: The cube has side 3 cm, so its volume is $3^3 = 27 \text{ cm}^3$, not $3 \times 3 = 9$ or 3×6 .

Marks: 1 mark for 7.8. 1 mark for 7.8×27 (27 must be the volume of the cube). 1 mark for 210.6

5 (2 marks)

Water flows from a tap at 12 litres per minute.

How long does it take to fill a tank of volume 0.9 m^3 ?

($1 \text{ m}^3 = 1000 \text{ litres}$)

Change the volume to litres: $0.9 \text{ m}^3 = 0.9 \times 1000 = 900 \text{ litres}$

Time = volume $\div$ rate

$$900 \div 12 = 75 \text{ minutes}$$

Answer: 75 minutes

Tip: Make the units match first. The rate is in litres, so change m^3 to litres.

Marks: 1 mark for $900 \div 12$. 1 mark for 75.

6 (3 marks)

The district of Ashby has 64 800 people living in it.

Its population density is 1350 people per km^2

$$\text{population density} = \frac{\text{population}}{\text{area}}$$

The district boundary is changed so that the area of Ashby increases by 12 km^2

The number of people living in Ashby does not change.

Work out the new population density of Ashby.

$$\text{Rearrange the formula: } \text{area} = \frac{\text{population}}{\text{population density}}$$

$$\text{Area} = 64\,800 \div 1350 = 48 \text{ km}^2$$

$$\text{New area} = 48 + 12 = 60 \text{ km}^2$$

$$\text{New population density} = 64\,800 \div 60 = 1080 \text{ people per km}^2$$

Answer: 1080

Tip: First find the original area. Adding 12 to the population density (or to 1350) is a common mistake.

Marks: 1 mark for $64\,800 \div 1350$. 1 mark for $64\,800 \div 60$. 1 mark for 1080.

7 (3 marks)

Sam makes a drink by mixing 50 cm^3 of fruit concentrate with some water.

The density of the fruit concentrate is 1.3 g/cm^3

The density of water is 1 g/cm^3

The density of the drink is 1.06 g/cm^3

Assume that the volume of the drink is the volume of the concentrate plus the volume of the water.

Work out the volume of water Sam uses.

Let the volume of water be $w \text{ cm}^3$

Mass of concentrate: $1.3 \times 50 = 65 \text{ g}$

Mass of water: $1 \times w = w \text{ g}$

Density of the drink = $\frac{\text{total mass}}{\text{total volume}}$, so $\frac{65 + w}{50 + w} = 1.06$

$$65 + w = 1.06(50 + w) = 53 + 1.06w$$

$$65 - 53 = 1.06w - w$$

$$12 = 0.06w$$

$$w = 12 \div 0.06 = 200$$

Answer: 200 cm^3

Tip: Density of a mixture = total mass $\div$ total volume. You cannot average the densities.

Marks: 1 mark for 65 (the mass of the concentrate). 1 mark for a correct equation such as $\frac{65+w}{50+w} = 1.06$. 1 mark for 200.

8 (3 marks)

Ali runs 400 m at 5 m/s and then 600 m at 4 m/s.
Work out his average speed for the whole run.

Time = distance $\div$ speed

First part: $400 \div 5 = 80$ s

Second part: $600 \div 4 = 150$ s

Total distance = $400 + 600 = 1000$ m

Total time = $80 + 150 = 230$ s

Average speed = $1000 \div 230 = 4.347 \dots$

To 3 significant figures: 4.35 m/s

Answer: 4.35 m/s (3 s.f.)

Tip: Average speed = total distance $\div$ total time. It is not the mean of 5 and 4.

Marks: 1 mark for 80 or 150. 1 mark for $1000 \div 230$. 1 mark for 4.35 (or better).

9 (3 marks)

Kofi drives 110 miles in 2 hours 15 minutes.
For the first part of the journey his average speed is 40 mph.
For the rest of the journey his average speed is 56 mph.
How many miles does Kofi drive at 40 mph?

2 hours 15 minutes = 2.25 hours

Let t be the time, in hours, at 40 mph. The time at 56 mph is $2.25 - t$

Total distance: $40t + 56(2.25 - t) = 110$

$40t + 126 - 56t = 110$

$126 - 16t = 110$, so $16t = 16$ and $t = 1$

Distance at 40 mph = $40 \times 1 = 40$ miles

(Check: $56 \times 1.25 = 70$ and $40 + 70 = 110$ ✓)

Answer: 40 miles

Tip: 15 minutes is 0.25 hours, not 0.15 hours. Write the two times in terms of one unknown.

Marks: 1 mark for 2.25 hours. 1 mark for a correct equation such as $40t + 56(2.25 - t) = 110$. 1 mark for 40.

10 (5 marks)

Maryam and Hamid live 20 km apart. A straight road joins their houses.

Maryam leaves her house at 10.10 am and cycles towards Hamid's house at a constant 15 km/h.

Hamid leaves his house at 10.30 am and walks towards Maryam's house at a constant 5 km/h.

(a) At what time do they meet?

Maryam starts 20 minutes before Hamid.

In 20 minutes = $\frac{1}{3}$ hour, Maryam cycles $15 \times \frac{1}{3} = 5$ km

At 10.30 am they are $20 - 5 = 15$ km apart.

They move towards each other, so the gap closes at $15 + 5 = 20$ km/h

Time to meet = $15 \div 20 = 0.75$ hours = 45 minutes

10.30 am + 45 minutes = 11.15 am

Answer: 11.15 am

Tip: When two people move towards each other, add their speeds to find how fast the gap between them closes.

Marks: 1 mark for 5 km. 1 mark for 15 km apart at 10.30 am. 1 mark for $15 \div 20$ (or 45 minutes). 1 mark for 11.15 am.

(b) How far from Maryam's house do they meet?

Maryam cycles from 10.10 am to 11.15 am: 65 minutes = $\frac{65}{60}$ hours

Distance = $15 \times \frac{65}{60} = 16.25$ km

(Check: Hamid walks $5 \times 0.75 = 3.75$ km and $16.25 + 3.75 = 20$ ✓)

Answer: 16.25 km
