

Composite Functions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

$$f(x) = x^2 \quad \text{and} \quad g(x) = 2x + 1$$

Circle the expression for $fg(x)$

Options: $(2x + 1)^2$, $2x^2 + 1$, $4x^2 + 1$, $x^2(2x + 1)$

$fg(x)$ means do g first, then f .

$$g(x) = 2x + 1$$

Now put $2x + 1$ into f , which squares its input:

$$f(2x + 1) = (2x + 1)^2$$

Answer: $(2x + 1)^2$

Tip: $fg(x)$ is not $f(x) \times g(x)$. You put $g(x)$ into f .

2 (7 marks)

$$f(x) = 2x + 5 \quad g(x) = x^2 - 3$$

(a) Work out $fg(2)$

$fg(2)$: work out $g(2)$ first.

$$g(2) = 2^2 - 3 = 1$$

Now put 1 into f :

$$f(1) = 2 \times 1 + 5 = 7$$

Answer: 7

Tip: Don't work out $f(2)$ and $g(2)$ separately and multiply them. The output of g becomes the input of f .

Marks: 1 mark for $g(2) = 1$. 1 mark for 7.

- (b) Work out $gf(-1)$

$gf(-1)$: work out $f(-1)$ first.

$$f(-1) = 2 \times (-1) + 5 = 3$$

Now put 3 into g :

$$g(3) = 3^2 - 3 = 6$$

Answer: 6

Marks: 1 mark for $f(-1) = 3$. 1 mark for 6.

- (c) Find an expression for $fg(x)$

$$fg(x) = f(x^2 - 3)$$

f multiplies by 2, then adds 5:

$$f(x^2 - 3) = 2(x^2 - 3) + 5$$

$$= 2x^2 - 6 + 5 = 2x^2 - 1$$

Answer: $2x^2 - 1$

- (d) Find an expression for $gf(x)$
Give your answer in expanded form.

$$gf(x) = g(2x + 5)$$

g squares its input, then subtracts 3: $(2x + 5)^2 - 3$

$$\text{Expand: } (2x + 5)(2x + 5) = 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25$$

Subtract 3: $4x^2 + 20x + 22$

Answer: $4x^2 + 20x + 22$

Tip: $(2x + 5)^2$ is not $4x^2 + 25$. Write it as $(2x + 5)(2x + 5)$ and expand fully.

Marks: 1 mark for $(2x + 5)^2 - 3$. 1 mark for $4x^2 + 20x + 22$.

3 (2 marks)

$g(x) = 20 - x$ and $h(x) = x^3$
Solve $gh(x) = 47$

$$gh(x) = g(x^3) = 20 - x^3$$

$$\text{Solve } 20 - x^3 = 47:$$

$$\text{Subtract 20: } -x^3 = 27$$

$$x^3 = -27$$

$$x = \sqrt[3]{-27} = -3$$

Answer: $x = -3$

Tip: The cube root of a negative number is negative: $\sqrt[3]{-27} = -3$, because $(-3)^3 = -27$.

Marks: 1 mark for $20 - x^3 = 47$. 1 mark for $x = -3$.

4 (3 marks)

$f(x) = 2x + 5$ and $g(x) = \frac{x}{3}$
Solve $fg(x) = 11$

$$fg(x) = f\left(\frac{x}{3}\right) = 2 \times \frac{x}{3} + 5 = \frac{2x}{3} + 5$$

$$\text{Solve } \frac{2x}{3} + 5 = 11:$$

$$\text{Subtract 5: } \frac{2x}{3} = 6$$

$$\text{Multiply by 3: } 2x = 18$$

$$x = 9$$

Answer: $x = 9$

Tip: You need to solve $fg(x) = 11$. Don't work out $fg(11)$ instead.

Marks: 1 mark for $\frac{2x}{3} + 5$. 1 mark for $\frac{2x}{3} = 6$. 1 mark for $x = 9$.

5 (3 marks)

$$f(x) = ax + b \quad \text{and} \quad g(x) = 2x - 3$$

a and b are constants.

$$gf(x) = 6x + 7$$

Work out the values of a and b .

$$gf(x) = g(ax + b)$$

$$= 2(ax + b) - 3$$

$$= 2ax + 2b - 3$$

This must be the same as $6x + 7$.

Compare the x terms: $2a = 6$, so $a = 3$

Compare the numbers: $2b - 3 = 7$, so $2b = 10$ and $b = 5$

Answer: $a = 3$, $b = 5$

Tip: $gf(x)$ means put $f(x)$ into g . Writing $a(2x - 3) + b$ is $fg(x)$, the wrong way round.

Marks: 1 mark for $2(ax + b) - 3$. 1 mark for $a = 3$. 1 mark for $b = 5$.

6 (5 marks)

$$f(x) = 2x - 3$$

$$g(x) = x^2 + x$$

(a) Find $gf(x)$

Give your answer in the form $ax^2 + bx + c$

$$gf(x) = g(2x - 3)$$

Replace each x in $x^2 + x$ with $(2x - 3)$:

$$(2x - 3)^2 + (2x - 3)$$

$$(2x - 3)^2 = 4x^2 - 12x + 9$$

$$\text{Add } 2x - 3: 4x^2 - 12x + 9 + 2x - 3 = 4x^2 - 10x + 6$$

Answer: $4x^2 - 10x + 6$

Marks: 1 mark for $(2x - 3)^2 + (2x - 3)$. 1 mark for $4x^2 - 10x + 6$.

(b) Find the values of x for which $gf(x) = g(x)$

$$4x^2 - 10x + 6 = x^2 + x$$

$$\text{Make one side 0: } 3x^2 - 11x + 6 = 0$$

$$\text{Factorise: } (3x - 2)(x - 3) = 0$$

$$3x - 2 = 0 \text{ gives } x = \frac{2}{3}$$

$$x - 3 = 0 \text{ gives } x = 3$$

Answer: $x = \frac{2}{3}$ or $x = 3$

Tip: Collect everything on one side so that it equals 0 before you factorise. A quadratic equation usually has two solutions: give both.

Marks: 1 mark for $3x^2 - 11x + 6 = 0$. 1 mark for factorising or using the formula. 1 mark for both answers.

7 (4 marks)

$$f(x) = x + 4 \text{ and } g(x) = x^2 - 6$$

$$\text{Solve } gf(x) = fg(x)$$

$$gf(x) = g(x + 4) = (x + 4)^2 - 6$$

$$= x^2 + 8x + 16 - 6 = x^2 + 8x + 10$$

$$fg(x) = f(x^2 - 6) = x^2 - 6 + 4 = x^2 - 2$$

$$\text{Set them equal: } x^2 + 8x + 10 = x^2 - 2$$

$$\text{The } x^2 \text{ terms cancel: } 8x + 10 = -2$$

$$8x = -12$$

$$x = -\frac{3}{2}$$

Answer: $x = -\frac{3}{2}$

Tip: Label each one ($gf(x) = \dots$ and $fg(x) = \dots$) so that you don't mix up the order.

Marks: 1 mark for $gf(x)$. 1 mark for $fg(x)$. 1 mark for $8x = -12$. 1 mark for $x = -\frac{3}{2}$.
