

Completing the Square - Harder

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (9 marks)

Write each expression in the form $a(x + b)^2 + c$

(a) $2x^2 + 12x + 5$

Take out the factor 2 from the x^2 and x terms:

$$2(x^2 + 6x) + 5$$

Complete the square inside the bracket: $x^2 + 6x = (x + 3)^2 - 9$

So we get $2[(x + 3)^2 - 9] + 5$

$$\begin{aligned} \text{Multiply out the square bracket: } & 2(x + 3)^2 - 18 + 5 \\ & = 2(x + 3)^2 - 13 \end{aligned}$$

Answer: $2(x + 3)^2 - 13$

Tip: The -9 is inside the bracket, so it gets multiplied by 2 as well: $2 \times (-9) = -18$.

Marks: 1 mark for $2(x^2 + 6x)$. 1 mark for $2(x + 3)^2$. 1 mark for the correct answer.

(b) $3x^2 - 6x + 1$

Take out the factor 3 from the x^2 and x terms:

$$3(x^2 - 2x) + 1$$

Complete the square inside the bracket: $x^2 - 2x = (x - 1)^2 - 1$

So we get $3[(x - 1)^2 - 1] + 1$

$$\text{Multiply out the square bracket: } 3(x - 1)^2 - 3 + 1$$

$$= 3(x - 1)^2 - 2$$

Answer: $3(x - 1)^2 - 2$

Marks: 1 mark for $3(x^2 - 2x)$. 1 mark for $3(x - 1)^2$. 1 mark for the correct answer.

(c) $2x^2 - 8x + 3$

Take out the factor 2 from the x^2 and x terms:

$$2(x^2 - 4x) + 3$$

Complete the square inside the bracket: $x^2 - 4x = (x - 2)^2 - 4$

So we get $2[(x - 2)^2 - 4] + 3$

Multiply out the square bracket: $2(x - 2)^2 - 8 + 3$

$$= 2(x - 2)^2 - 5$$

Answer: $2(x - 2)^2 - 5$

Marks: 1 mark for $2(x^2 - 4x)$. 1 mark for $2(x - 2)^2$. 1 mark for the correct answer.

2 (4 marks)

(a) Write $5 + 4x - x^2$ in the form $p - (x + q)^2$

Take out -1 from the x^2 and x terms:

$$5 + 4x - x^2 = -(x^2 - 4x) + 5$$

Complete the square inside the bracket: $x^2 - 4x = (x - 2)^2 - 4$

So we get $-[(x - 2)^2 - 4] + 5$

Multiply out the square bracket: $-(x - 2)^2 + 4 + 5$

$$= 9 - (x - 2)^2$$

Answer: $9 - (x - 2)^2$

Tip: Taking out -1 changes the sign of the x term: $-x^2 + 4x = -(x^2 - 4x)$. Then $-(-4) = +4$.

Marks: 1 mark for $(x - 2)^2$. 1 mark for the correct answer.

- (b) Hence write down the maximum value of $5 + 4x - x^2$ and the value of x where it occurs.

$$5 + 4x - x^2 = 9 - (x - 2)^2$$

$(x - 2)^2$ is never negative, so we are always taking away 0 or more from 9.

So the biggest value is 9.

This happens when $(x - 2)^2 = 0$, so when $x = 2$.

Answer: Maximum value is 9, when $x = 2$

Marks: 1 mark for 9. 1 mark for $x = 2$ (follow-through from part (a)).

3 (4 marks)

By completing the square, solve $2x^2 + 4x - 3 = 0$
Give your answers to 2 decimal places.

Take out the factor 2: $2(x^2 + 2x) - 3 = 0$

Complete the square: $x^2 + 2x = (x + 1)^2 - 1$

$$2[(x + 1)^2 - 1] - 3 = 0$$

$$2(x + 1)^2 - 2 - 3 = 0$$

$$2(x + 1)^2 - 5 = 0$$

Add 5 to both sides: $2(x + 1)^2 = 5$

Divide both sides by 2: $(x + 1)^2 = 2.5$

Square root both sides: $x + 1 = \pm\sqrt{2.5}$

Subtract 1: $x = -1 \pm \sqrt{2.5}$

$$x = -1 + 1.581... = 0.581... \text{ or } x = -1 - 1.581... = -2.581...$$

Answer: $x = 0.58$ and $x = -2.58$

Tip: Divide by 2 **before** you square root. Then remember $\pm$ to get both answers.

Marks: 1 mark for $2(x+1)^2$. 1 mark for $2(x+1)^2 - 5$. 1 mark for $x = -1 \pm \sqrt{2.5}$. 1 mark for both answers.

4 (3 marks)

Write $3x^2 - 18x + 11$ in the form $d(x+e)^2 + f$ where d , e and f are integers.

Take out the factor 3 from the x^2 and x terms:

$$3(x^2 - 6x) + 11$$

Complete the square inside the bracket: $x^2 - 6x = (x-3)^2 - 9$

So we get $3[(x-3)^2 - 9] + 11$

Multiply out the square bracket: $3(x-3)^2 - 27 + 11$

$$= 3(x-3)^2 - 16$$

Answer: $3(x-3)^2 - 16$

Marks: 1 mark for $3(x^2 - 6x)$. 1 mark for $3(x-3)^2$. 1 mark for the correct answer.

5 (6 marks)

This question is about the expression $2x^2 + 6x + 7$

(a) Write $2x^2 + 6x + 7$ in the form $a(x+b)^2 + c$

Take out the factor 2 from the x^2 and x terms: $2(x^2 + 3x) + 7$

Complete the square: halve 3 to get $\frac{3}{2}$.

$$x^2 + 3x = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$$

So we get $2\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$

Multiply out: $2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + 7$

$$-4.5 + 7 = 2.5$$

$$\text{So } 2x^2 + 6x + 7 = 2(x + 1.5)^2 + 2.5$$

Answer: $2(x + 1.5)^2 + 2.5$

Tip: When you multiply out $2\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right]$, the $-\frac{9}{4}$ is multiplied by 2 too. Forgetting this gives $c = 4.75$, which is wrong.

Marks: 1 mark for $2(x^2 + 3x)$ or $a = 2$. 1 mark for $2(x + 1.5)^2$. 1 mark for $+2.5$.

(b) Hence, solve $2x^2 + 6x + 7 = 15$

$$\text{Use part (a): } 2(x + 1.5)^2 + 2.5 = 15$$

$$\text{Subtract 2.5 from both sides: } 2(x + 1.5)^2 = 12.5$$

$$\text{Divide both sides by 2: } (x + 1.5)^2 = 6.25$$

$$\text{Square root both sides: } x + 1.5 = \pm 2.5$$

$$x = -1.5 + 2.5 = 1 \text{ or } x = -1.5 - 2.5 = -4$$

Answer: $x = 1$ or $x = -4$

Tip: Remember the $\pm$ when you square root. Leaving it out loses the solution $x = -4$.

Marks: 1 mark for $(x + 1.5)^2 = 6.25$ or $x + 1.5 = \pm 2.5$. 1 mark for both solutions.

(c) Explain why the equation $2x^2 + 6x + 7 = 2$ has no solutions.

$$\text{From part (a), } 2x^2 + 6x + 7 = 2(x + 1.5)^2 + 2.5$$

$(x + 1.5)^2$ can never be negative, so $2(x + 1.5)^2 + 2.5$ is always at least 2.5

2 is less than 2.5, so $2x^2 + 6x + 7$ can never equal 2.

Answer: The smallest value of $2x^2 + 6x + 7$ is 2.5, so it can never equal 2

Tip: Refer to your completed square: a squared bracket is never negative, so the expression is never less than 2.5.