

Circle Theorems

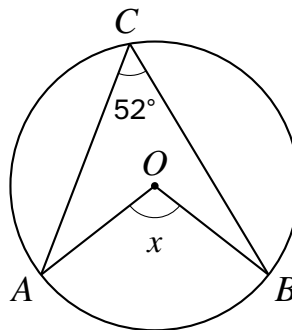
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A, B and C are points on a circle, centre O.



Not drawn accurately

Work out the size of angle x .

Give a reason for your answer.

Angle x is at the centre and 52° is at the circumference.

Both angles stand on the same arc, AB .

$$x = 2 \times 52 = 104^\circ$$

Reason: the angle at the centre is twice the angle at the circumference.

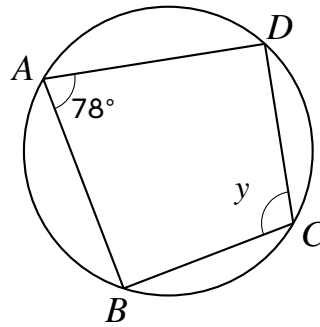
Answer: $x = 104^\circ$

Tip: The angle at the centre is the bigger one. Double the angle at the circumference; do not halve it.

Marks: 1 mark for 104. 1 mark for the reason.

2 (2 marks)

$ABCD$ is a cyclic quadrilateral.



Not drawn accurately

Work out the size of angle y .

Give a reason for your answer.

Angles y and 78° are opposite each other in the cyclic quadrilateral.

$$y = 180 - 78 = 102^\circ$$

Reason: opposite angles of a cyclic quadrilateral add up to 180° .

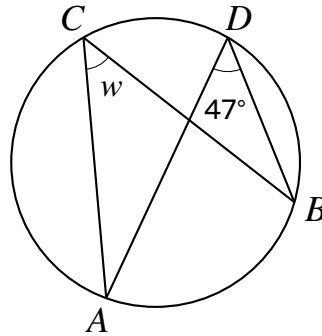
Answer: $y = 102^\circ$

Tip: Opposite angles in a cyclic quadrilateral add up to 180° . They are not equal.

Marks: 1 mark for 102. 1 mark for the reason.

3 (2 marks)

A , B , C and D are points on a circle.



Not drawn accurately

Work out the size of angle w .

Give a reason for your answer.

Angles ACB and ADB both stand on the chord AB .

They are on the same side of AB , so they are in the same segment.

$$w = 47^\circ$$

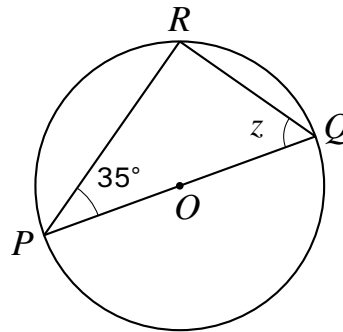
Reason: angles in the same segment are equal.

Answer: $w = 47^\circ$

Marks: 1 mark for 47. 1 mark for the reason.

4 (3 marks)

P , Q and R are points on a circle, centre O .



Not drawn accurately

PQ is a diameter of the circle.

Work out the size of angle z .

Give reasons for your answer.

Angle $PRQ = 90^\circ$

Reason: the angle in a semicircle is 90° .

$$z = 180 - 90 - 35 = 55^\circ$$

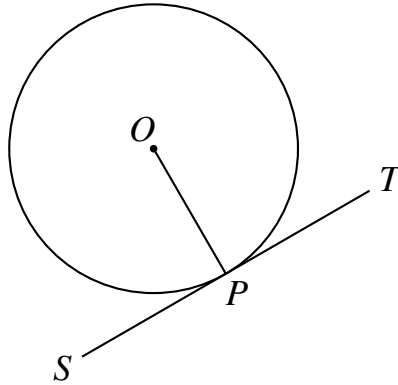
Reason: angles in a triangle add up to 180° .

Answer: $z = 55^\circ$

Marks: 1 mark for angle $PRQ = 90^\circ$. 1 mark for 55. 1 mark for both reasons.

5 (1 mark)

ST is a tangent to the circle, centre O , at the point P .



Not drawn accurately

What is the size of angle OPT ?

Circle your answer.

Options: 45° , 60° , 90° , 180°

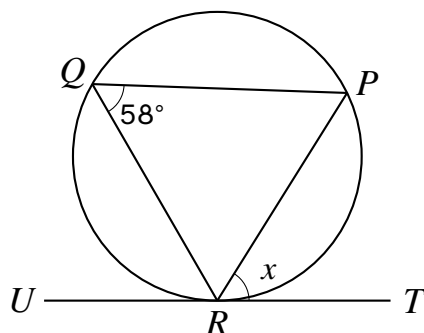
OP is a radius and ST is the tangent at P .

A tangent is perpendicular to the radius at the point where it touches the circle.

Answer: 90°

6 (2 marks)

P , Q and R are points on a circle.



Not drawn accurately

UT is the tangent to the circle at R .

Angle $PQR = 58^\circ$

Work out the size of angle x .

Give a reason for your answer.

Angle x (angle PRT) is between the tangent RT and the chord RP .

Angle PQR is in the alternate segment: it stands on the chord RP , on the other side of RP from the angle x .

So $x = 58^\circ$

Reason: alternate segment theorem. The angle between a tangent and a chord is equal to the angle in the alternate segment.

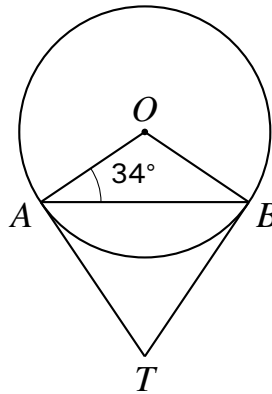
Answer: $x = 58^\circ$

Tip: Do not assume lines are parallel unless the question says so. Here the reason is the alternate segment theorem.

Marks: 1 mark for 58. 1 mark for the reason.

7 (3 marks)

A and B are points on a circle, centre O .



Not drawn accurately

TA and TB are tangents to the circle.

Angle $OAB = 34^\circ$

Work out the size of angle ATB .

Give reasons for your answer.

$OA = OB$ (radii), so triangle OAB is isosceles.

Angle $OBA = 34^\circ$ (base angles of an isosceles triangle are equal)

Angle $AOB = 180 - 34 - 34 = 112^\circ$ (angles in a triangle add up to 180°)

Angle $OAT = \text{angle } OBT = 90^\circ$

Reason: a tangent is perpendicular to the radius.

Angle $ATB = 360 - 90 - 90 - 112 = 68^\circ$

Reason: angles in a quadrilateral add up to 360° .

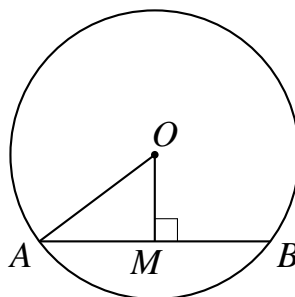
Answer: Angle $ATB = 68^\circ$

Tip: Another way: angle $TAB = 90 - 34 = 56^\circ$, and $TA = TB$ (tangents from a point are equal), so angle $ATB = 180 - 2 \times 56 = 68^\circ$.

Marks: 1 mark for angle $AOB = 112^\circ$ or angle $TAB = 56^\circ$. 1 mark for 68. 1 mark for all the reasons.

8 (3 marks)

A and B are points on a circle, centre O .



Not drawn accurately

OM is perpendicular to the chord AB .

The radius of the circle is 10 cm. $AB = 16$ cm

Work out the length of OM .

The perpendicular from the centre to a chord bisects the chord.

So $AM = 16 \div 2 = 8$ cm

OA is a radius, so $OA = 10$ cm

Triangle OAM has a right angle at M .

Pythagoras: $OM^2 = 10^2 - 8^2 = 100 - 64 = 36$

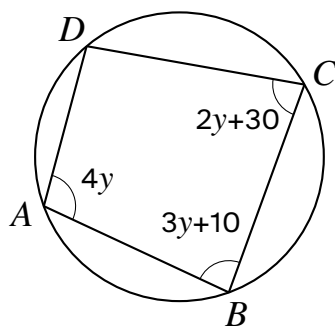
$OM = \sqrt{36} = 6$

Answer: 6 cm

Marks: 1 mark for $AM = 8$. 1 mark for $10^2 - 8^2$. 1 mark for 6.

9 (3 marks)

A, B, C and D are points on a circle.



Not drawn accurately

$ABCD$ is a cyclic quadrilateral.

Angle $BAD = 4y$, angle $ABC = 3y + 10$ and angle $BCD = 2y + 30$

Work out the size of angle ADC .

Angles BAD and BCD are opposite angles, so they add up to 180° .

$$4y + 2y + 30 = 180$$

$$6y = 150, \text{ so } y = 25$$

$$\text{Angle } ABC = 3 \times 25 + 10 = 85^\circ$$

Angles ABC and ADC are opposite angles, so angle $ADC = 180 - 85 = 95^\circ$

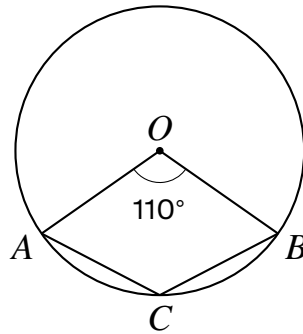
Answer: Angle $ADC = 95^\circ$

Tip: Only the **opposite** angles of a cyclic quadrilateral add up to 180° . Angles BAD and ABC are next to each other, so do not add those to make 180° .

Marks: 1 mark for $4y + 2y + 30 = 180$. 1 mark for $y = 25$. 1 mark for 95.

10 (1 mark)

A , B and C are points on a circle, centre O .



Not drawn accurately

Imani is trying to work out the size of angle ACB .

Here is her working.

$$\text{Angle } ACB = 110 \div 2 = 55^\circ$$

The angle at the centre is twice the angle at the circumference.

Make one criticism of Imani's working.

C is on the minor arc AB (the short arc).

So angle ACB does not stand on the same arc as the 110° angle at the centre.

The reflex angle AOB is $360 - 110 = 250^\circ$.

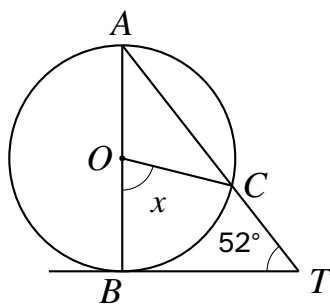
Angle $ACB = 250 \div 2 = 125^\circ$, not 55° .

Answer: C is on the minor arc, so she should halve the reflex angle: angle $ACB = 250 \div 2 = 125^\circ$

Tip: Check which arc the angle stands on. The angle at the centre must stand on the same arc as the angle at the circumference.

11 (3 marks)

A , B and C lie on a circle with centre O .



Not drawn accurately

The line AB is a diameter.

The tangent at B meets the line AC , extended beyond C , at the point T .

Work out the size of angle x .

Angle $ABT = 90^\circ$

(A tangent is perpendicular to the radius, and OB is part of the diameter AB .)

In triangle ABT : angle $BAT = 180 - 90 - 52 = 38^\circ$

(Angles in a triangle add up to 180° .)

Angle $BAC = 38^\circ$ is at the circumference and x is at the centre, both on the arc BC .

$$x = 2 \times 38 = 76^\circ$$

(The angle at the centre is twice the angle at the circumference.)

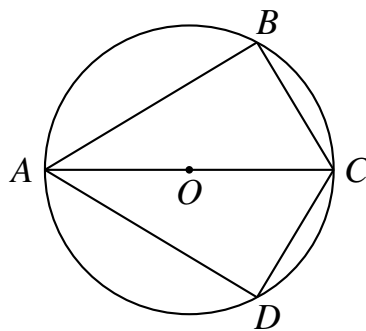
Answer: $x = 76^\circ$

Tip: Angle x is at the centre, so it is double the angle at A , not half of it.

Marks: 1 mark for angle $ABT = 90^\circ$. 1 mark for angle $BAT = 38^\circ$. 1 mark for 76.

12 (4 marks)

A, B, C and D lie on a circle with centre O .



Not drawn accurately

The line AC passes through O .

Angle BAC = angle DAC

Prove that $AB = AD$.

AC passes through the centre O , so AC is a diameter.

Angle $ABC = 90^\circ$

(The angle in a semicircle is 90° .)

Angle $ADC = 90^\circ$

(The angle in a semicircle is 90° .)

Angle BAC = angle DAC (given)

AC is a side of both triangles ABC and ADC .

So triangles ABC and ADC are congruent (AAS).

So the corresponding sides AB and AD are equal: $AB = AD$.

Tip: Give a reason for every step, and name the congruence condition you use (here AAS).

Name angles with three letters, e.g. angle ABC , not “angle B ”.

Marks: 1 mark for angle $ABC = 90^\circ$ or angle $ADC = 90^\circ$ with the reason. 1 mark for both right angles. 1 mark for using AC as a common side with the equal angles. 1 mark for a complete proof that the triangles are congruent, so $AB = AD$.