

Bearings with Trigonometry

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

C is North West of *D*.

Write down the bearing of *D* from *C*.

If *C* is North West of *D*, then *D* is South East of *C*.

Bearings are measured clockwise from North.

East is 090° and South is 180° .

South East is halfway between them: 135°

Answer: 135°

Tip: Read the question carefully: you need the direction of *D* **from** *C*, so stand at *C*. North West would be 315° , the bearing of *C* from *D*.

2 (1 mark)

The bearing of *Y* from *X* is 065°

Circle the bearing of *X* from *Y*.

Options: 115° , 245° , 295° , 065°

The return bearing points in the opposite direction.

Add 180° : $65 + 180 = 245$

Answer: 245°

Tip: For a back bearing, add 180° if the bearing is less than 180° . Subtract 180° if it is more.

3 (2 marks)

X is due West of Y .

Z is due South of Y .

Angle $YXZ = 35^\circ$

Work out the bearing of Z from X .

Draw a sketch: Y is due East of X , and Z is below Y .

From X , the direction of Y is due East, a bearing of 090° .

Z is 35° further round clockwise from the direction of Y (towards South).

Bearing of Z from $X = 90 + 35 = 125^\circ$

Answer: 125°

Tip: Always draw a North line at the point you are measuring **from** (here X). $090 - 35 = 055^\circ$ is a common wrong answer: Z is South of the East line, so you add.

Marks: 1 mark for a correct sketch or for using 090° . 1 mark for 125.

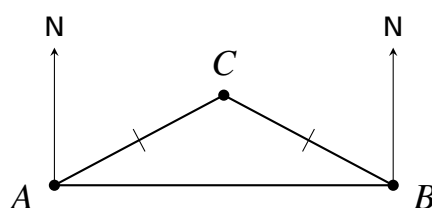
4 (3 marks)

A , B and C are three buoys.

B is due East of A .

$AC = BC$

The bearing of C from A is 062°



Not drawn accurately

Work out the bearing of C from B .

From A , B is due East (090°) and C is on a bearing of 062° .

So angle $CAB = 90 - 62 = 28^\circ$

$AC = BC$, so triangle ABC is isosceles and angle $CBA = 28^\circ$ too.

From B , the direction of A is due West, a bearing of 270° .

C is 28° further round clockwise (towards North).

Bearing of C from $B = 270 + 28 = 298^\circ$

Answer: 298°

Tip: Draw a North line at B . C is North West of B , so the bearing must be between 270° and 360° .

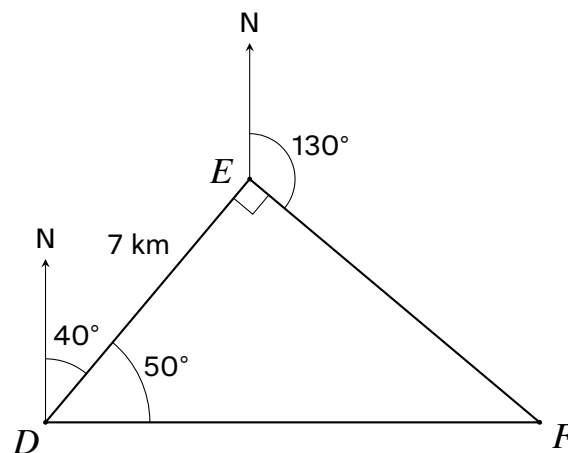
Marks: 1 mark for 28° . 1 mark for angle $CBA = 28^\circ$ or for using 270° . 1 mark for 298 .

5 (7 marks)

A drone flies 7 km from its base D to a point E on a bearing of 040° .
It then flies on a bearing of 130° until it reaches a point F that is due East of D .

(a) Work out the distance from E to F .

Sketch the flight with a North line at D and at E .



The bearing of D from E is $040 + 180 = 220^\circ$.

The drone leaves E on a bearing of 130° , so angle $DEF = 220 - 130 = 90^\circ$

F is due East of D , so angle $EDF = 90 - 40 = 50^\circ$

In the right-angled triangle DEF : $\tan 50^\circ = \frac{EF}{7}$

$EF = 7 \times \tan 50^\circ = 8.342 \dots$

$$= 8.34 \text{ km (3 s.f.)}$$

Answer: 8.34 km

Tip: Find the angles inside the triangle first. The bearings 040° and 130° are not angles of the triangle.

Marks: 1 mark for angle $DEF = 90^\circ$. 1 mark for angle $EDF = 50^\circ$. 1 mark for $7 \tan 50$. 1 mark for 8.34.

- (b) Work out the distance from D to F .

DF is the hypotenuse of triangle DEF .

$$\cos 50^\circ = \frac{7}{DF}$$

$$DF = \frac{7}{\cos 50^\circ} = 10.89 \dots$$

$$= 10.9 \text{ km (3 s.f.)}$$

(Or use Pythagoras with 7 and 8.342 ...)

Answer: 10.9 km

Tip: The hypotenuse must be the longest side. If your answer is less than 8.34 km, check your working.

Marks: 1 mark for a correct method, e.g. $\cos 50 = \frac{7}{DF}$. 1 mark for $7 \div \cos 50$. 1 mark for 10.9.

6 (4 marks)

Point Q is 12 km due East of point P .

Point R is 9 km from P on a bearing of 120°

Work out the distance QR .

Due East is a bearing of 090° .

$$\text{So angle } QPR = 120 - 90 = 30^\circ$$

Triangle PQR is not right-angled, so use the cosine rule:

$$QR^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \times \cos 30^\circ$$

$$QR^2 = 225 - 187.06 \dots = 37.93 \dots$$

$$QR = \sqrt{37.93 \dots} = 6.158 \dots$$

$$= 6.16 \text{ (3 s.f.)}$$

Answer: 6.16 km

Tip: Only use Pythagoras when you know there is a right angle. Here you need the cosine rule.

Marks: 1 mark for 30° . 1 mark for substituting into the cosine rule. 1 mark for $QR^2 = 37.9 \dots$ 1 mark for 6.16.

7 (5 marks)

Two boats leave a harbour H at the same time.

Boat A sails on a bearing of 030° at 12 km/h.

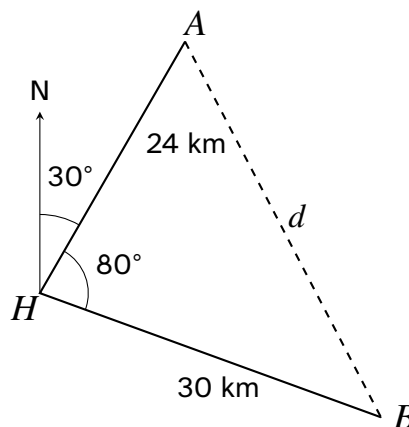
Boat B sails on a bearing of 110° at 15 km/h.

Work out the distance between the boats after 2 hours.

Distance = speed $\times$ time

Boat A : $12 \times 2 = 24$ km. Boat B : $15 \times 2 = 30$ km

Angle between the two directions at $H = 110 - 30 = 80^\circ$



Let d be the distance between the boats. Use the cosine rule:

$$d^2 = 24^2 + 30^2 - 2 \times 24 \times 30 \times \cos 80^\circ$$

$$d^2 = 1476 - 250.0 \dots = 1225.9 \dots$$

$$d = 35.01 \dots$$

$$= 35.0 \text{ km (3 s.f.)}$$

Answer: 35.0 km

Tip: The triangle is not right-angled, so Pythagoras ($\sqrt{24^2 + 30^2} = 38.4$) is wrong here. Use the cosine rule with the 80° angle.

Marks: 1 mark for 24 and 30. 1 mark for 80° . 1 mark for substituting into the cosine rule. 1 mark for $d^2 = 1226$ (or better). 1 mark for 35.0.

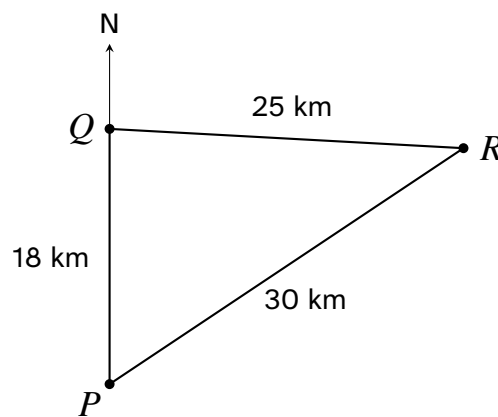
8 (6 marks)

P , Q and R are three towns.

Q is 18 km due North of P .

R is 30 km from P and 25 km from Q .

R is to the East of the line PQ .



Not drawn accurately

- (a) Work out the bearing of R from P .
Give your answer to the nearest degree.

Q is due North of P , so the bearing of R from P is the size of angle QPR .

Use the cosine rule for angle QPR :

$$\cos QPR = \frac{18^2 + 30^2 - 25^2}{2 \times 18 \times 30} = \frac{599}{1080}$$

$$\text{Angle } QPR = \cos^{-1} \left(\frac{599}{1080} \right) = 56.3 \dots^\circ$$

Bearing of R from $P = 056^\circ$ (nearest degree)

Answer: 056°

Tip: Put the side opposite the angle you want (here $QR = 25$) as the one you subtract:
 $18^2 + 30^2 - 25^2$.

Marks: 1 mark for a correct cosine rule statement for angle QPR . 1 mark for $\frac{599}{1080}$ or 56.3 1 mark for 056 .

- (b) Work out the bearing of R from Q .
Give your answer to the nearest degree.

Find angle PQR with the cosine rule:

$$\cos PQR = \frac{18^2 + 25^2 - 30^2}{2 \times 18 \times 25} = \frac{49}{900}$$

$$\text{Angle } PQR = \cos^{-1}\left(\frac{49}{900}\right) = 86.87 \dots^\circ$$

From Q , P is due South, a bearing of 180° .

R is $86.87 \dots^\circ$ anticlockwise from South (towards East).

Bearing of R from $Q = 180 - 86.87 \dots = 93.12 \dots$

$= 093^\circ$ (nearest degree)

Answer: 093°

Tip: Angle PQR is not the bearing. Draw a North line at Q : the angle from North to QR is 180° minus angle PQR .

Marks: 1 mark for $\cos PQR = \frac{49}{900}$ (or angle PQR from a correct method, e.g. the sine rule). 1 mark for 86.9° . 1 mark for 093 .