

Area of a Triangle Using Sine

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

In triangle ABC , a and b are two sides and C is the angle between them.
Which of these is the area of the triangle?

Circle your answer.

Options: $ab \sin C$, $\frac{1}{2}ab \sin C$, $\frac{1}{2}ab \cos C$, $\frac{1}{2}ab$

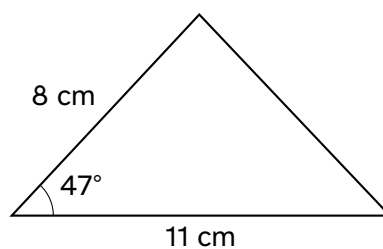
$$\text{Area} = \frac{1}{2}ab \sin C$$

C must be the angle between the sides a and b .

Answer: $\frac{1}{2}ab \sin C$

2 (2 marks)

Here is a triangle.



Not drawn accurately

Work out the area of the triangle.

Give your answer to 3 significant figures.

The 47° angle is between the 8 cm and 11 cm sides.

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 8 \times 11 \times \sin 47^\circ = 32.179 \dots$$

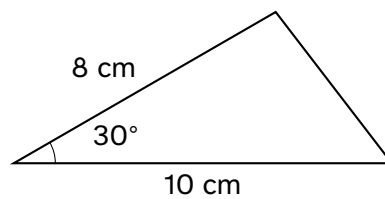
Answer: 32.2 cm²

Tip: $\frac{1}{2} \times \text{base} \times \text{height}$ only works with the perpendicular height. Here the sides are not perpendicular, so use $\frac{1}{2}ab \sin C$.

Marks: 1 mark for $\frac{1}{2} \times 8 \times 11 \times \sin 47^\circ$. 1 mark for 32.2.

3 (2 marks)

Here is a triangle.



Not drawn accurately

$$\sin 30^\circ = 0.5$$

Show that the area of the triangle is 20 cm²

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 8 \times 10 \times \sin 30^\circ$$

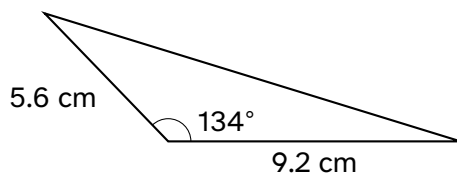
$$\frac{1}{2} \times 8 \times 10 = 40$$

$$40 \times 0.5 = 20 \text{ cm}^2 \quad \checkmark$$

Marks: 1 mark for $\frac{1}{2} \times 8 \times 10 \times \sin 30^\circ$. 1 mark for $40 \times 0.5 = 20$.

4 (2 marks)

Here is a triangle.



Not drawn accurately

Work out the area of the triangle.

Give your answer to 3 significant figures.

The 134° angle is between the 5.6 cm and 9.2 cm sides.

$$\text{Area} = \frac{1}{2} \times 5.6 \times 9.2 \times \sin 134^\circ$$

$$\text{Area} = 18.53 \dots$$

Answer: 18.5 cm²

Marks: 1 mark for $\frac{1}{2} \times 5.6 \times 9.2 \times \sin 134^\circ$. 1 mark for 18.5.

5 (3 marks)

A patio is in the shape of a triangle PQR .

$PQ = 3.2$ m and $PR = 4.5$ m. Angle QPR is obtuse.

The area of the patio is 6.1 m².

Work out the size of angle QPR .

Give your answer to 1 decimal place.

Angle QPR is between sides PQ and PR .

$$\frac{1}{2} \times 3.2 \times 4.5 \times \sin P = 6.1$$

$$7.2 \sin P = 6.1$$

$$\sin P = \frac{6.1}{7.2} = 0.8472 \dots$$

$$\sin^{-1}(0.8472 \dots) = 57.91 \dots^\circ, \text{ but this is acute.}$$

$$\text{The obtuse angle with the same sine is } 180 - 57.91 \dots = 122.08 \dots$$

$$= 122.1^\circ \text{ (1 d.p.)}$$

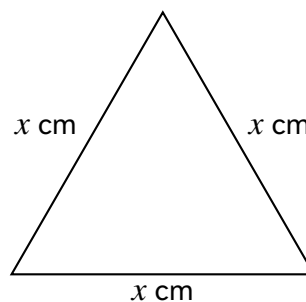
Answer: 122.1°

Tip: Your calculator's $\sin^{-1}$ always gives an acute angle. If the angle is obtuse, subtract that answer from 180° .

Marks: 1 mark for $\frac{1}{2} \times 3.2 \times 4.5 \times \sin P = 6.1$. 1 mark for $\sin P = 0.847 \dots$ or 57.9° . 1 mark for 122.1 .

6 (3 marks)

Here is an equilateral triangle.



Not drawn accurately

The area of the triangle is 50 cm^2 .

Work out the length of one side.

Give your answer to 3 significant figures.

Call the side length x .

Every angle in an equilateral triangle is 60° .

$$\frac{1}{2} \times x \times x \times \sin 60^\circ = 50$$

$$x^2 = \frac{100}{\sin 60^\circ} = 115.47 \dots$$

$$x = \sqrt{115.47 \dots} = 10.74 \dots$$

$$= 10.7 \text{ cm (3 s.f.)}$$

Answer: 10.7 cm

Tip: Don't stop at x^2 . Remember to take the square root at the end.

Marks: 1 mark for $\frac{1}{2} \times x \times x \times \sin 60^\circ = 50$. 1 mark for $x^2 = 115.4 \dots$ 1 mark for 10.7.

7 (3 marks)

Work out the area of a regular hexagon with sides of 6 cm.
Give your answer to 3 significant figures.

Join the centre to each corner.

This splits the hexagon into 6 equilateral triangles.

Each triangle has sides of 6 cm and angles of 60° .

$$\text{Area of one triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 60^\circ = 15.58 \dots$$

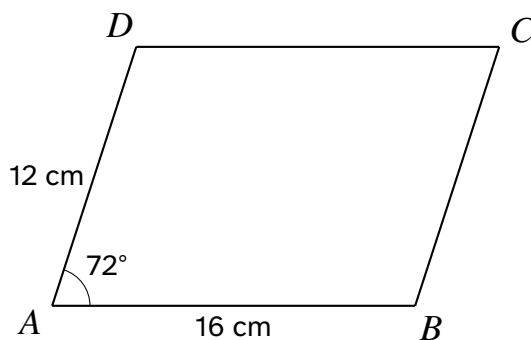
$$\text{Area of hexagon} = 6 \times 15.58 \dots = 93.53 \dots$$

Answer: 93.5 cm²

Marks: 1 mark for $\frac{1}{2} \times 6 \times 6 \times \sin 60^\circ$. 1 mark for multiplying by 6. 1 mark for 93.5.

8 (3 marks)

$ABCD$ is a parallelogram.



Not drawn accurately

Work out the area of the parallelogram.

Give your answer to 3 significant figures.

Draw the diagonal BD .

It splits the parallelogram into two congruent triangles.

$$\text{Area of triangle } ABD = \frac{1}{2} \times 16 \times 12 \times \sin 72^\circ = 91.30 \dots$$

$$\text{Area of parallelogram} = 2 \times 91.30 \dots = 182.60 \dots$$

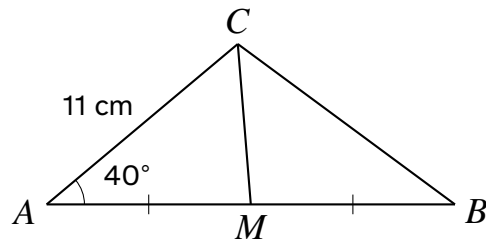
Answer: 183 cm²

Tip: The area is not 16×12 , because 12 cm is not the perpendicular height. Use two triangles, or area = $ab \sin C$.

Marks: 1 mark for $\frac{1}{2} \times 16 \times 12 \times \sin 72^\circ$. 1 mark for multiplying by 2. 1 mark for 183.

9 (3 marks)

ABC is a triangle.



Not drawn accurately

$AB = 18$ cm, $AC = 11$ cm and angle $BAC = 40^\circ$

M is the midpoint of AB .

Work out the area of triangle AMC .

M is the midpoint of AB , so $AM = 18 \div 2 = 9$ cm

The 40° angle is between AM and AC .

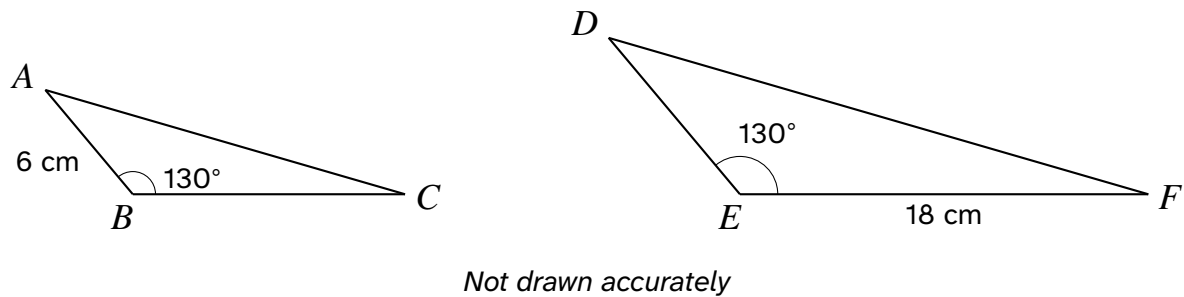
Area of $AMC = \frac{1}{2} \times 9 \times 11 \times \sin 40^\circ = 31.81 \dots$

Answer: 31.8 cm²

Marks: 1 mark for $AM = 9$. 1 mark for $\frac{1}{2} \times 9 \times 11 \times \sin 40^\circ$. 1 mark for 31.8.

10 (4 marks)

Here are two triangles.



Triangles ABC and DEF are similar.

$AB = 6$ cm, angle $ABC = \text{angle } DEF = 130^\circ$ and $EF = 18$ cm

The area of triangle ABC is 27.58 cm^2

Work out the area of triangle DEF .

First find BC , using the area of triangle ABC .

$$\frac{1}{2} \times 6 \times BC \times \sin 130^\circ = 27.58$$

$$3 \sin 130^\circ \times BC = 27.58$$

$$BC = \frac{27.58}{3 \sin 130^\circ} = 12.00 \dots \text{ cm}$$

EF matches BC , so the length scale factor $= 18 \div 12 = 1.5$

Area scale factor $= 1.5^2 = 2.25$

Area of $DEF = 27.58 \times 2.25 = 62.055$

Answer: 62.1 cm^2 (62 to 62.1 is fine)

Tip: For areas, square the length scale factor: multiply by $1.5^2 = 2.25$, not by 1.5.

Marks: 1 mark for $\frac{1}{2} \times 6 \times BC \times \sin 130^\circ = 27.58$. 1 mark for $BC = 12$. 1 mark for $27.58 \times (18 \div 12)^2$. 1 mark for an answer from 62 to 62.1.