

Area and Perimeter of Compound Shapes with Circles

Solutions

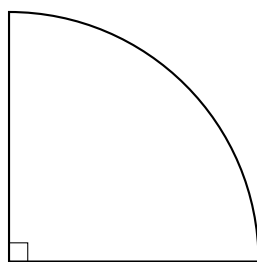
For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Here is a quarter circle.

The area of the quarter circle is $16\pi \text{ cm}^2$.



Not drawn accurately

(a) Work out the radius of the quarter circle.

$$\text{Area of a quarter circle} = \frac{1}{4}\pi r^2$$

$$\frac{1}{4}\pi r^2 = 16\pi$$

$$r^2 = 16 \times 4 = 64$$

$$r = \sqrt{64} = 8$$

Answer: 8 cm

Tip: Remember the $\frac{1}{4}$. If you use $\pi r^2 = 16\pi$ you get $r = 4$, which is the radius of a full circle with this area.

Marks: 1 mark for $\frac{1}{4}\pi r^2 = 16\pi$ or $r^2 = 64$. 1 mark for 8.

- (b) Work out the perimeter of the quarter circle.
Give your answer in the form $a + b\pi$ where a and b are integers.

The perimeter is the curved arc plus the two straight radii.

$$\text{Arc} = \frac{1}{4} \times 2 \times \pi \times 8 = 4\pi$$

$$\text{Straight edges} = 8 + 8 = 16$$

$$\text{Perimeter} = 16 + 4\pi$$

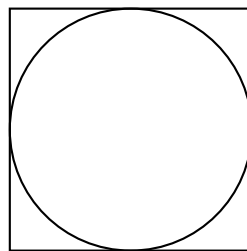
Answer: $16 + 4\pi$ cm

Tip: Don't forget the two straight edges ($8 + 8$). The arc on its own is not the perimeter.

Marks: 1 mark for $\frac{1}{4} \times 2\pi \times 8$ (or 4π). 1 mark for $16 + 4\pi$.

2 (3 marks)

A circle fits exactly inside a square.
The circle touches all four sides of the square.



Not drawn accurately

The area of the circle is $49\pi \text{ cm}^2$.
Work out the perimeter of the square.

$$\text{Area of circle} = \pi r^2 = 49\pi$$

$$r^2 = 49, \text{ so } r = 7 \text{ cm}$$

The circle touches all four sides, so the side of the square is the diameter of the circle.

$$\text{Side of the square} = 2 \times 7 = 14 \text{ cm}$$

Perimeter of the square = $4 \times 14 = 56$ cm

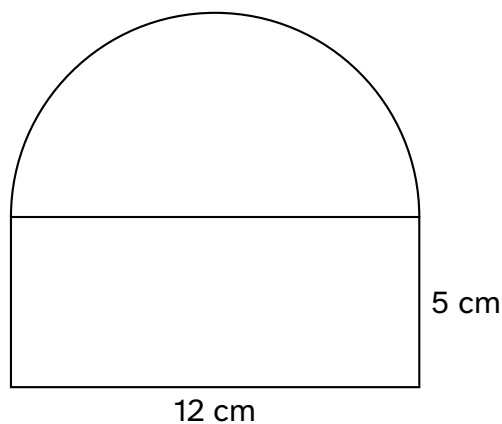
Answer: 56 cm

Tip: The side of the square is the **diameter**, not the radius. Using 7 as the side gives 28, a common wrong answer.

Marks: 1 mark for $r = 7$. 1 mark for a side of 14. 1 mark for 56.

3 (6 marks)

The shape is made from a rectangle and a semicircle.



Not drawn accurately

- (a) Work out the area of the shape.
Give your answer to 3 significant figures.

Split the shape into a rectangle and a semicircle.

$$\text{Rectangle} = 12 \times 5 = 60 \text{ cm}^2$$

The semicircle has diameter 12 cm, so radius 6 cm.

$$\text{Semicircle} = \frac{1}{2} \times \pi \times 6^2 = 56.548 \dots \text{ cm}^2$$

$$\text{Total} = 60 + 56.548 \dots = 116.548 \dots$$

$$= 117 \text{ (3 s.f.)}$$

Answer: 117 cm²

Marks: 1 mark for the rectangle area (60). 1 mark for the semicircle area (56.5 ...). 1 mark for 117.

- (b) Work out the perimeter of the shape.
Give your answer to 3 significant figures.

The perimeter is the outside edge only.

The top of the rectangle is inside the shape, so do not count it.

$$\text{Semicircular arc} = \frac{1}{2} \times \pi \times 12 = 18.849 \dots \text{ cm}$$

$$\begin{aligned} \text{Perimeter} &= 12 + 5 + 5 + 18.849 \dots = 40.849 \dots \\ &= 40.8 \text{ (3 s.f.)} \end{aligned}$$

Answer: 40.8 cm

Tip: Use πd (not πr^2) for the curved edge and halve it for a semicircle. Do not include the 12 cm line where the semicircle joins the rectangle.

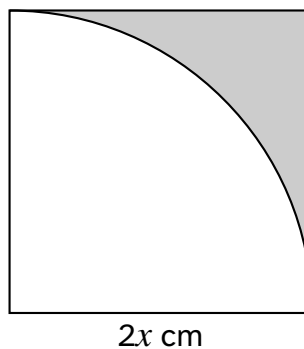
Marks: 1 mark for the arc length (18.8 ...). 1 mark for adding $12 + 5 + 5$. 1 mark for 40.8.

4 (4 marks)

The diagram shows a square of side $2x$ cm.

A quarter circle of radius $2x$ cm is drawn inside the square, with its centre at one corner of the square.

The shaded area is 50 cm^2 .



Not drawn accurately

Work out the value of x .

Give your answer to 3 significant figures.

Shaded area = square – quarter circle

$$\text{Square} = 2x \times 2x = 4x^2$$

The radius of the quarter circle is the side of the square, $2x$.

$$\text{Quarter circle} = \frac{1}{4} \times \pi \times (2x)^2 = \frac{1}{4} \times \pi \times 4x^2 = \pi x^2$$

$$\text{Shaded area} = 4x^2 - \pi x^2 = x^2(4 - \pi)$$

$$x^2(4 - \pi) = 50$$

$$x^2 = \frac{50}{4 - \pi} = 58.24 \dots$$

$$x = \sqrt{58.24 \dots} = 7.631 \dots$$

$$x = 7.63 \text{ (3 s.f.)}$$

Answer: $x = 7.63$

Tip: Square the whole of $2x$: $(2x)^2 = 4x^2$, not $2x^2$. Keep $4 - \pi$ exact on your calculator; rounding it to 0.86 first gives 7.62.

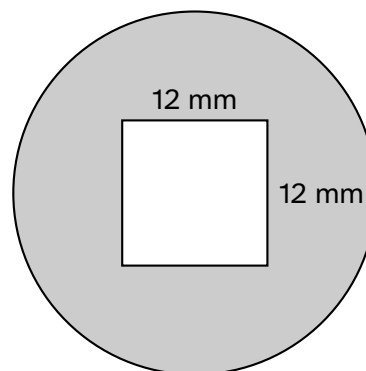
Marks: 1 mark for $4x^2$ and πx^2 (or $\frac{1}{4}\pi(2x)^2$). 1 mark for $4x^2 - \pi x^2 = 50$. 1 mark for $x^2 = \frac{50}{4-\pi}$ (or 58.2...). 1 mark for 7.63.

5 (4 marks)

A square hole is cut from the centre of a circular metal disc.

The diameter of the disc is 30 mm.

The hole is a square with sides of length 12 mm.



Not drawn accurately

Work out the area of the hole as a percentage of the area of the disc.

Give your answer to 1 decimal place.

The radius of the disc is $30 \div 2 = 15$ mm.

$$\text{Area of the disc} = \pi \times 15^2 = 706.858 \dots \text{ mm}^2$$

$$\text{Area of the hole} = 12 \times 12 = 144 \text{ mm}^2$$

$$\text{Percentage} = \frac{144}{706.858 \dots} \times 100$$

$$= 20.371 \dots$$

$$= 20.4\% \text{ (1 d.p.)}$$

Answer: 20.4%

Tip: Halve the diameter before using πr^2 . Using $\pi \times 30^2$ gives an area four times too big.

Marks: 1 mark for $\pi \times 15^2$ (or 706.8 ...). 1 mark for 144. 1 mark for $\frac{144}{706.8 \dots} \times 100$. 1 mark for 20.4.

6 (5 marks)

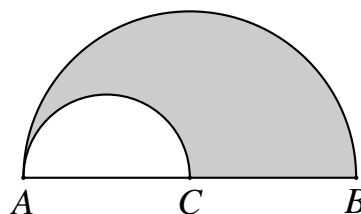
The diagram shows two semicircles.

AB is the diameter of the large semicircle.

AC is the diameter of the small semicircle.

C is the midpoint of AB .

The area of the shaded region is $24\pi \text{ cm}^2$.



Not drawn accurately

Work out the perimeter of the shaded region.

Give your answer in the form $a\pi + b$ where a and b are integers.

Let the radius of the large semicircle be $r \text{ cm}$.

C is the midpoint of AB , so $AC = r$ and the small semicircle has radius $\frac{r}{2}$.

$$\text{Large semicircle area} = \frac{1}{2} \pi r^2$$

$$\text{Small semicircle area} = \frac{1}{2} \pi \left(\frac{r}{2}\right)^2 = \frac{1}{8} \pi r^2$$

$$\text{Shaded area} = \frac{1}{2} \pi r^2 - \frac{1}{8} \pi r^2 = \frac{3}{8} \pi r^2$$

$$\frac{3}{8}\pi r^2 = 24\pi, \text{ so } r^2 = 24 \times \frac{8}{3} = 64$$

$$r = 8, \text{ so } AB = 16 \text{ cm and } AC = CB = 8 \text{ cm}$$

$$\text{Large arc} = \frac{1}{2} \times \pi \times 16 = 8\pi$$

$$\text{Small arc} = \frac{1}{2} \times \pi \times 8 = 4\pi$$

The perimeter is the large arc, the small arc and the straight line CB .

$$\text{Perimeter} = 8\pi + 4\pi + 8 = 12\pi + 8$$

Answer: $12\pi + 8 \text{ cm}$

Tip: The small semicircle's radius is half the large one's, so its area is a **quarter** of the large semicircle's area, not a half. For the perimeter, include the straight edge CB but not AC , which is not on the edge of the shaded region.

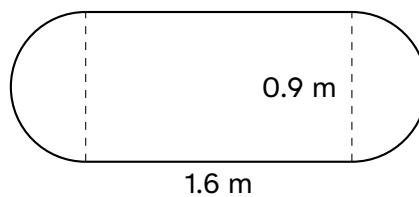
Marks: 1 mark for an expression for the shaded area, e.g. $\frac{1}{2}\pi r^2 - \frac{1}{8}\pi r^2$. 1 mark for $r = 8$ (or $AB = 16$). 1 mark for 8π or 4π . 1 mark for including $CB = 8$. 1 mark for $12\pi + 8$.

7 (7 marks)

A table top is a rectangle with a semicircle at each end.

The straight edges are each 1.6 m long.

The table top is 0.9 m wide.



Not drawn accurately

(a) A strip of trim is fixed all the way round the edge of the table top.

The trim is sold in rolls.

Each roll is 2.5 m long.

How many rolls of trim are needed?

You must show your working.

The two curved ends make one full circle of diameter 0.9 m.

$$\text{Curved edges} = \pi \times 0.9 = 2.827 \dots \text{ m}$$

$$\text{Straight edges} = 2 \times 1.6 = 3.2 \text{ m}$$

$$\text{Total length of edge} = 3.2 + 2.827 \dots = 6.027 \dots \text{ m}$$

$$\text{Rolls} = 6.027 \dots \div 2.5 = 2.41 \dots$$

2 rolls are not enough, so round **up**.

Answer: 3

Tip: The two semicircles together make one whole circle, so use πd once. Then round the number of rolls **up**.

Marks: 1 mark for $\pi \times 0.9$ (or two semicircle arcs). 1 mark for a total edge of 6.03 1 mark for 3.

- (b) A smaller table top is also a rectangle with a semicircle at each end.

It is 0.9 m wide.

The total perimeter of the smaller table top is 5.5 m.

Work out the area of the smaller table top.

Give your answer to 3 significant figures.

The two curved ends make one full circle of diameter 0.9 m.

$$\text{Curved edges} = \pi \times 0.9 = 2.827 \dots \text{ m}$$

$$\text{Two straight edges} = 5.5 - 2.827 \dots = 2.672 \dots \text{ m}$$

$$\text{Each straight edge} = 2.672 \dots \div 2 = 1.336 \dots \text{ m}$$

$$\text{Rectangle} = 1.336 \dots \times 0.9 = 1.202 \dots \text{ m}^2$$

The two semicircles make one circle of radius $0.9 \div 2 = 0.45 \text{ m}$.

$$\text{Circle} = \pi \times 0.45^2 = 0.636 \dots \text{ m}^2$$

$$\text{Total} = 1.202 \dots + 0.636 \dots = 1.838 \dots$$

$$= 1.84 \text{ (3 s.f.)}$$

Answer: 1.84 m²

Tip: Take off the curved edges first, then halve what is left: there are **two** straight edges. Use the diameter 0.9 for the curved length but the radius 0.45 for the area.

Marks: 1 mark for $5.5 - \pi \times 0.9$. 1 mark for a straight edge of 1.336 1 mark for $1.336 \dots \times 0.9 + \pi \times 0.45^2$. 1 mark for 1.84.