

Arcs, Sectors and Segments

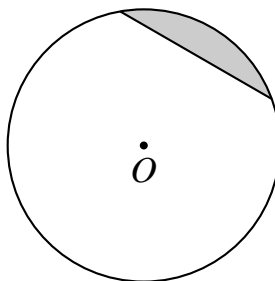
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The diagram shows a circle, centre O .



Not drawn accurately

Circle the name of the shaded region.

Options: arc, chord, sector, segment

The shaded region is between a chord (a straight line) and an arc.

This region is called a segment.

Answer: segment

Tip: A **sector** is a “pizza slice” with two radii. A **segment** is cut off by a chord. These two are often mixed up.

2 (2 marks)

A sector has radius 9 cm and angle 80°

Work out the length of the arc.

Give your answer in terms of π

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{80}{360} \times 2 \times \pi \times 9$$

$$= \frac{2}{9} \times 18\pi$$

$$= 4\pi$$

Answer: 4π cm

Tip: Arc length uses the circumference, $2\pi r$. Area uses πr^2 . Don't mix them up.

Marks: 1 mark for $\frac{80}{360} \times 2 \times \pi \times 9$. 1 mark for 4π .

3 (2 marks)

A sector of a circle has radius 6 cm and angle 150°

Work out the area of the sector. Give your answer in terms of π .

$$\text{Sector area} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{150}{360} \times \pi \times 6^2$$

$$\text{Simplify the fraction first: } \frac{150}{360} = \frac{5}{12}$$

$$= \frac{5}{12} \times 36\pi = 15\pi$$

Answer: 15π cm²

Marks: 1 mark for $\frac{150}{360} \times \pi \times 6^2$. 1 mark for 15π .

4 (2 marks)

A sector has radius 7 cm and arc length 11 cm.

Work out the angle of the sector.

Give your answer to the nearest degree.

Let the angle be θ .

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$\frac{\theta}{360} \times 2 \times \pi \times 7 = 11$$

$$\frac{\theta}{360} \times 14\pi = 11$$

$$\theta = \frac{11 \times 360}{14\pi} = 90.03 \dots$$

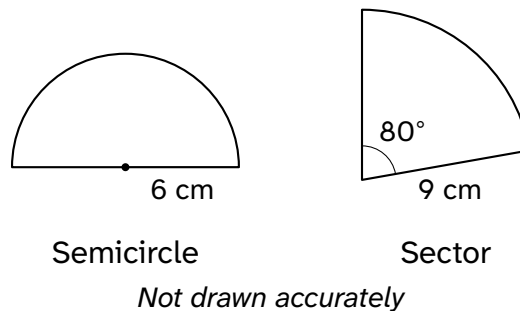
To the nearest degree: 90°

Answer: 90°

Marks: 1 mark for $\frac{\theta}{360} \times 14\pi = 11$. 1 mark for 90.

5 (2 marks)

The diagram shows a semicircle and a sector of a different circle.
The semicircle has radius 6 cm.



Tomasz says,

“The semicircle and the sector have the same area.”

Is Tomasz correct?

Tick a box.

Show working to support your answer.

Work out both areas.

Semicircle: $\frac{1}{2} \times \pi \times 6^2 = \frac{1}{2} \times 36\pi = 18\pi$

Sector: $\frac{80}{360} \times \pi \times 9^2 = \frac{2}{9} \times 81\pi = 18\pi$

Both areas are $18\pi \text{ cm}^2$ (56.5 cm^2 to 3 s.f.), so they are the same.

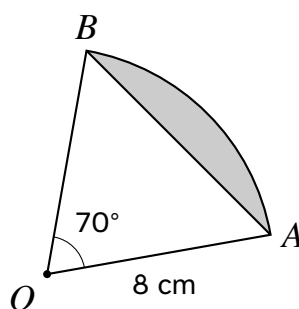
Answer: Yes (both are $18\pi \text{ cm}^2$)

Tip: A semicircle is half a circle, so its area is $\frac{1}{2}\pi r^2$. It is a sector with angle 180° .

Marks: 1 mark for either area. 1 mark for both areas and ticking Yes.

6 (6 marks)

OAB is a sector of a circle, centre O , radius 8 cm.



Not drawn accurately

(a) Work out the length of the arc AB .

$$\begin{aligned}\text{Arc length} &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{70}{360} \times 2 \times \pi \times 8 \\ &= 9.773 \dots \\ &= 9.77 \text{ (3 s.f.)}\end{aligned}$$

Answer: 9.77 cm

Marks: 1 mark for $\frac{70}{360} \times 2\pi \times 8$. 1 mark for 9.77.

(b) Work out the area of the shaded segment.

Segment = sector OAB – triangle OAB

$$\text{Sector} = \frac{70}{360} \times \pi \times 8^2 = 39.095 \dots$$

Use area of a triangle = $\frac{1}{2}ab \sin C$:

$$\text{Triangle} = \frac{1}{2} \times 8 \times 8 \times \sin 70^\circ = 30.070 \dots$$

$$\text{Segment} = 39.095 \dots - 30.070 \dots = 9.025 \dots$$

$$= 9.03 \text{ (3 s.f.)}$$

Answer: 9.03 cm²

Tip: Triangle OAB is not right-angled, so use $\frac{1}{2}ab \sin C$, not $\frac{1}{2} \times \text{base} \times \text{height}$ with 8 and 8. Keep full calculator values until the end.

Marks: 1 mark for the sector area. 1 mark for the triangle area using $\frac{1}{2}ab \sin C$. 1 mark for subtracting. 1 mark for 9.03.

7 (3 marks)

The area of a sector is 12π cm². The radius is 6 cm.

Work out the perimeter of the sector.

Give your answer in the form $a + b\pi$

Let the angle of the sector be θ .

$$\frac{\theta}{360} \times \pi \times 6^2 = 12\pi$$

$$\frac{\theta}{360} \times 36\pi = 12\pi$$

$$\frac{\theta}{360} = \frac{12}{36} = \frac{1}{3}, \text{ so } \theta = 120^\circ$$

$$\text{Arc length} = \frac{1}{3} \times 2 \times \pi \times 6 = 4\pi$$

$$\text{Perimeter} = \text{two radii} + \text{arc} = 6 + 6 + 4\pi = 12 + 4\pi$$

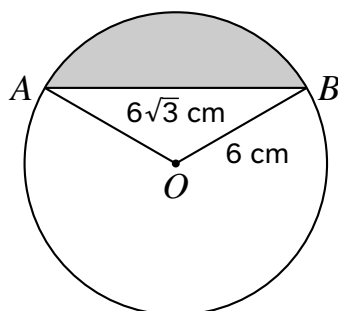
Answer: 12 + 4 π cm

Tip: The perimeter of a sector is the arc **plus the two radii**. Don't forget the 6 + 6.

Marks: 1 mark for finding the angle (120°) or the fraction $\frac{1}{3}$. 1 mark for the arc length 4π . 1 mark for $12 + 4\pi$.

8 (4 marks)

A and B are points on a circle, centre O , radius 6 cm.
The chord AB has length $6\sqrt{3}$ cm.



Not drawn accurately

Work out the area of the shaded segment.

Give your answer in the form $a\pi - b\sqrt{3}$ where a and b are integers.

Shaded segment = sector OAB – triangle OAB

First find angle AOB .

Triangle OAB is isosceles, so the line from O to the midpoint of AB is perpendicular to AB .

Half of AB is $3\sqrt{3}$

$$\sin\left(\frac{1}{2} \text{ angle } AOB\right) = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}, \text{ so } \frac{1}{2} \text{ angle } AOB = 60^\circ$$

Angle $AOB = 120^\circ$

$$\text{Sector} = \frac{120}{360} \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi$$

$$\text{Triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 120^\circ = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$$

$$\text{Shaded area} = 12\pi - 9\sqrt{3}$$

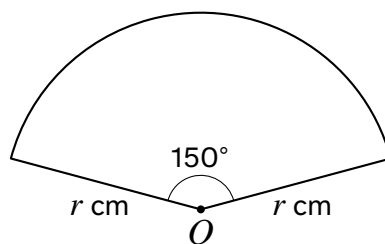
Answer: $12\pi - 9\sqrt{3} \text{ cm}^2$

Tip: Learn the exact values $\sin 60^\circ = \sin 120^\circ = \frac{\sqrt{3}}{2}$. You need them to give the answer in surd form. You can also find angle AOB with the cosine rule.

Marks: 1 mark for angle $AOB = 120^\circ$. 1 mark for 12π . 1 mark for $9\sqrt{3}$. 1 mark for $12\pi - 9\sqrt{3}$.

9 (4 marks)

The diagram shows a sector of a circle, centre O , radius r cm.



Not drawn accurately

The perimeter of the sector is 40 cm.

Work out the value of r .

Give your answer in the form $\frac{a}{b + c\pi}$ where a , b and c are integers.

$$\text{Arc length} = \frac{150}{360} \times 2\pi r = \frac{5}{12} \times 2\pi r = \frac{5\pi r}{6}$$

$$\text{Perimeter} = \text{two radii} + \text{arc} = 2r + \frac{5\pi r}{6}$$

$$\text{So } 2r + \frac{5\pi r}{6} = 40$$

$$\text{Multiply both sides by 6: } 12r + 5\pi r = 240$$

$$\text{Factorise: } r(12 + 5\pi) = 240$$

$$\text{Divide by } (12 + 5\pi): r = \frac{240}{12 + 5\pi}$$

$$\text{Answer: } r = \frac{240}{12 + 5\pi}$$

Tip: Include both radii in the perimeter. Then collect the r terms and factorise so that r appears only once.

Marks: 1 mark for the arc length $\frac{150}{360} \times 2\pi r$. 1 mark for the equation $2r + \frac{5\pi r}{6} = 40$. 1 mark for factorising $r(12 + 5\pi)$. 1 mark for the correct answer.