

Angles in Parallel Lines and Polygons - Higher

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

What is the sum of the interior angles of a hexagon?

Circle your answer.

Options: 360° , 540° , 720° , 1080°

A hexagon has 6 sides.

It can be split into $6 - 2 = 4$ triangles from one vertex.

Sum of interior angles = $4 \times 180 = 720^\circ$

Answer: 720°

Tip: Use $(n - 2) \times 180^\circ$. 360° is the sum of the **exterior** angles of any polygon.

2 (2 marks)

The interior angle of a regular polygon is 156°

How many sides does the polygon have?

Interior angle + exterior angle = 180° (angles on a straight line).

Exterior angle = $180 - 156 = 24^\circ$

The exterior angles add up to 360° .

Number of sides = $360 \div 24 = 15$

Answer: 15

Tip: Work with the exterior angle. $360 \div 156$ does not give the number of sides.

Marks: 1 mark for $360 \div (180 - 156)$ or $360 \div 24$. 1 mark for 15.

3 (2 marks)

The sum of the interior angles of a polygon is 1980°

How many sides does the polygon have?

For a polygon with n sides, the interior angles add up to $(n - 2) \times 180^\circ$.

So $(n - 2) \times 180 = 1980$

Divide both sides by 180:

$$n - 2 = 11$$

$$n = 13$$

Answer: 13

Tip: $1980 \div 180 = 11$ is $n - 2$, not n . Remember to add 2 at the end.

Marks: 1 mark for $1980 \div 180$ (or 11). 1 mark for 13.

4 (2 marks)

Is it possible for a regular polygon to have an interior angle of 130° ? Tick a box.

You must show your working.

$$\text{Exterior angle} = 180 - 130 = 50^\circ$$

$$\text{Number of sides} = 360 \div 50 = 7.2$$

The number of sides must be a whole number.

So it is not possible.

Answer: No, because $360 \div 50 = 7.2$, which is not a whole number.

Marks: 1 mark for $360 \div 50$. 1 mark for No, with 7.2 given as the reason.

5 (3 marks)

The angles of a pentagon are x , $2x$, $2x + 10$, 110° and 120°
Work out the size of the largest angle.

A pentagon has 5 sides.

$$\text{Angle sum} = (5 - 2) \times 180 = 540^\circ$$

$$\text{Add up the angles: } x + 2x + 2x + 10 + 110 + 120 = 5x + 240$$

$$\text{So } 5x + 240 = 540$$

$$5x = 300$$

$$x = 60$$

The angles are 60° , 120° , 130° , 110° and 120° .

The largest is $2x + 10 = 130^\circ$.

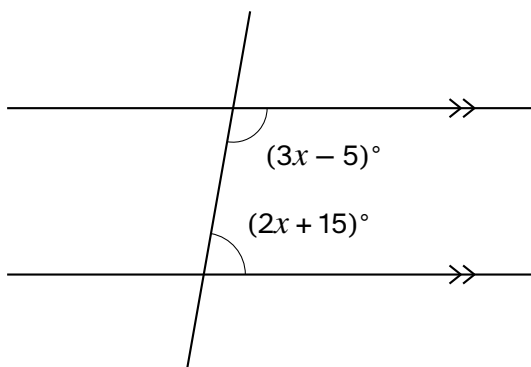
Answer: 130°

Tip: The angles of a polygon only add up to 360° for a quadrilateral. Also, the question asks for the largest angle, not x .

Marks: 1 mark for 540. 1 mark for $5x + 240 = 540$. 1 mark for 130.

6 (3 marks)

The diagram shows two parallel lines crossed by a transversal.



Not drawn accurately

Work out the value of x .

Give reasons for your answer.

The two angles are between the parallel lines, on the same side of the transversal.

So they are co-interior angles.

Co-interior angles add up to 180° .

$$(2x + 15) + (3x - 5) = 180$$

$$5x + 10 = 180$$

$$5x = 170$$

$$x = 34$$

Answer: $x = 34$ (co-interior angles add up to 180°)

Tip: Use the correct names in reasons: “co-interior”, not “C-angles”.

Marks: 1 mark for $(2x + 15) + (3x - 5) = 180$. 1 mark for $x = 34$. 1 mark for the reason “co-interior angles add up to 180° ”.

7 (3 marks)

Rhian says,

“The more sides a regular polygon has, the bigger each exterior angle is.”

- (a) Work out the size of an exterior angle of a regular polygon with 12 sides.

The exterior angles of any polygon add up to 360° .

$$360 \div 12 = 30^\circ$$

Answer: 30°

- (b) Work out the size of an exterior angle of a regular polygon with 18 sides.

$$360 \div 18 = 20^\circ$$

Answer: 20°

- (c) Is Rhian correct?

Tick one box and give a reason for your answer.

The polygon with more sides (18) has the smaller exterior angle ($20^\circ < 30^\circ$).

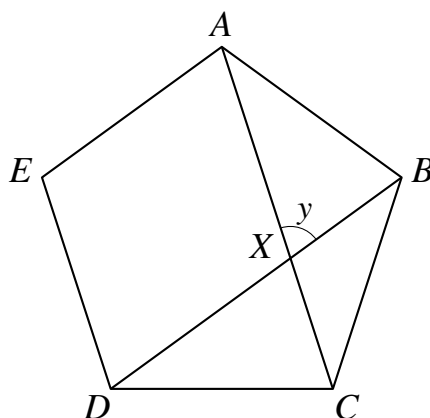
The 360° is shared between more angles, so each one is smaller.

So Rhian is not correct.

Answer: No: with more sides, each exterior angle is smaller (360° is shared between more angles)

8 (3 marks)

Here is a regular pentagon.



Not drawn accurately

ABCDE is a regular pentagon.

The diagonals *AC* and *BD* meet at *X*.

Work out the size of angle *y*.

Exterior angle of a regular pentagon = $360 \div 5 = 72^\circ$, so each interior angle = $180 - 72 = 108^\circ$

Triangle *ABC* is isosceles ($AB = BC$), so angle $BCA = (180 - 108) \div 2 = 36^\circ$

Triangle *BCD* is isosceles ($BC = CD$), so angle $CBD = (180 - 108) \div 2 = 36^\circ$

In triangle *BXC*: angle $BXC = 180 - 36 - 36 = 108^\circ$

AXC is a straight line, so $y = 180 - 108 = 72^\circ$

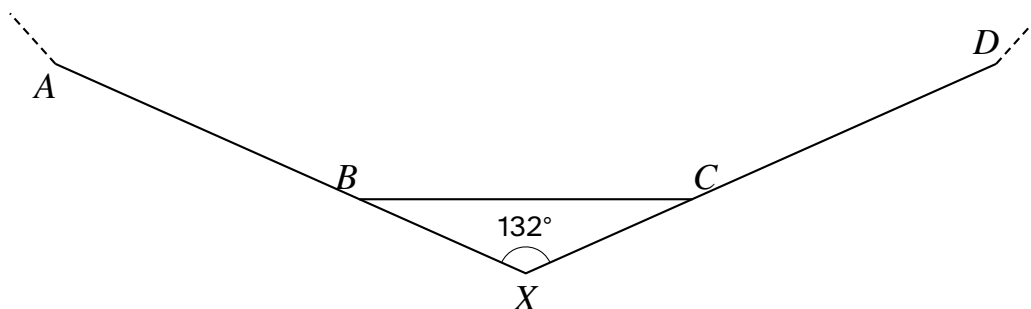
Answer: 72°

Tip: Write each angle you find on the diagram, or say which angle it is (e.g. "angle $BCA = 36^\circ$ "). A number with no label may not get the marks.

Marks: 1 mark for 108. 1 mark for 36 (angle *BCA* or angle *CBD*). 1 mark for 72.

9 (3 marks)

AB , BC and CD are three sides of a regular polygon.
The sides AB and DC are extended to meet at X .



Not drawn accurately

Angle $BXC = 132^\circ$

Work out the number of sides of the polygon.

Angles XBC and XCB are both exterior angles of the regular polygon, so they are equal.

Angles in a triangle add up to 180° .

In triangle BXC : each exterior angle = $(180 - 132) \div 2 = 24^\circ$

Number of sides = $360 \div 24 = 15$

Answer: 15

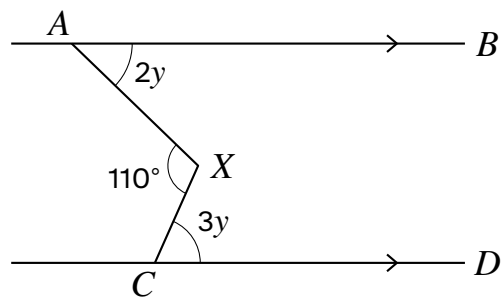
Tip: Angle XBC is on a straight line with the interior angle ABC , so it is an **exterior** angle of the polygon.

Marks: 1 mark for recognising angle XBC (or XCB) as an exterior angle. 1 mark for $(180 - 132) \div 2 = 24$. 1 mark for 15.

10 (3 marks)

AB and CD are parallel lines.

A , X and C are joined to make the angles shown.



Not drawn accurately

Angle $AXC = 110^\circ$

Work out the size of angle DCX .

Draw a line through X parallel to AB and CD .

This splits angle AXC into two parts.

The top part equals angle $BAX = 2y$ (alternate angles are equal).

The bottom part equals angle $DCX = 3y$ (alternate angles are equal).

$$\text{So } 2y + 3y = 110$$

$$5y = 110$$

$$y = 22$$

$$\text{Angle } DCX = 3 \times 22 = 66^\circ$$

Answer: 66°

Tip: Adding a third parallel line through X turns this into two pairs of alternate angles.

Remember the question asks for angle DCX , not y .

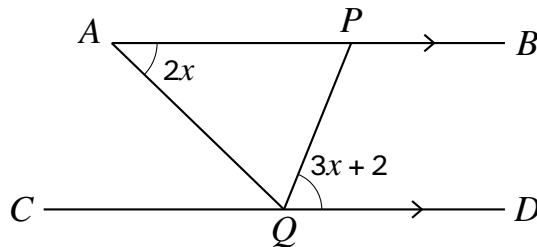
Marks: 1 mark for $2y + 3y = 110$ (or a correct use of alternate angles). 1 mark for $y = 22$. 1 mark for 66.

11 (4 marks)

AB and CD are straight, parallel lines.

P is a point on AB . Q is a point on CD .

$AP = AQ$



Not drawn accurately

Work out the value of x .

$AP = AQ$, so triangle APQ is isosceles.

Angle $PAQ = 2x$, so the two base angles add up to $180 - 2x$.

$$\text{Angle } APQ = \frac{180 - 2x}{2} = 90 - x$$

Angle $PQD = \text{angle } APQ$, because alternate angles are equal.

$$\text{So } 3x + 2 = 90 - x$$

$$\text{Add } x \text{ to both sides: } 4x + 2 = 90$$

$$4x = 88$$

$$x = 22$$

Answer: 22

Tip: The equal angles in an isosceles triangle are the ones opposite the equal sides. Here they are at P and Q , not at A .

Marks: 1 mark for $\frac{180-2x}{2}$ or $90 - x$. 1 mark for $3x + 2 = 90 - x$. 1 mark for $4x = 88$. 1 mark for 22.