

3D Pythagoras and Trigonometry

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A cuboid measures 3 cm by 4 cm by 12 cm.

Work out the length of the longest straight line that can be drawn inside the cuboid.

The longest line goes from one corner to the opposite corner (the space diagonal).

3D Pythagoras: $d^2 = a^2 + b^2 + c^2$

$$d^2 = 3^2 + 4^2 + 12^2 = 9 + 16 + 144 = 169$$

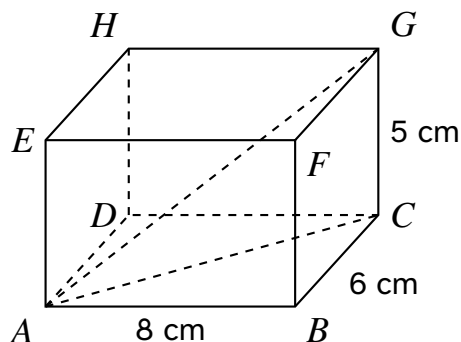
$$d = \sqrt{169} = 13$$

Answer: 13 cm

Marks: 1 mark for $3^2 + 4^2 + 12^2$. 1 mark for 13.

2 (6 marks)

The diagram shows a cuboid $ABCDEFGH$.



Not drawn accurately

- (a) Work out the length AG .

Step 1: find the base diagonal AC .

Triangle ABC has a right angle at B .

$$AC^2 = 8^2 + 6^2 = 100, \text{ so } AC = 10 \text{ cm}$$

Step 2: triangle ACG has a right angle at C (CG is vertical).

$$AG^2 = AC^2 + CG^2 = 100 + 5^2 = 125$$

$$AG = \sqrt{125} = 11.18 \dots$$

Answer: 11.2 cm

Marks: 1 mark for $8^2 + 6^2$. 1 mark for $AC^2 + 5^2$ (or $8^2 + 6^2 + 5^2$). 1 mark for 11.2.

- (b) Work out the angle between AG and the base $ABCD$.

C is directly below G , so AC is the “shadow” of AG on the base.

The angle we want is angle GAC .

In triangle ACG : opposite = $CG = 5$, adjacent = $AC = 10$

$$\tan \theta = \frac{5}{10} = 0.5$$

$$\theta = \tan^{-1}(0.5) = 26.56 \dots$$

Answer: 26.6°

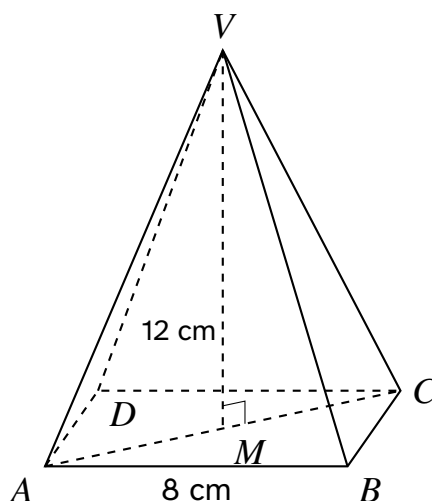
Tip: The angle between a line and a plane is between the line and its shadow on the plane. Here that is angle GAC .

Marks: 1 mark for finding the correct angle (GAC). 1 mark for $\tan \theta = \frac{5}{10}$ (or another correct ratio). 1 mark for 26.6.

3 (5 marks)

$VABCD$ is a pyramid with a square base of side 8 cm.

M is the centre of the base and V is vertically above M . $VM = 12$ cm.



Not drawn accurately

- (a) Work out the length VA .

Step 1: find AM , which is half the diagonal of the base.

$$AC = \sqrt{8^2 + 8^2} = \sqrt{128} = 11.31 \dots$$

$$AM = \frac{1}{2} \times 11.31 \dots = 5.656 \dots \text{ cm}$$

Step 2: triangle VMA has a right angle at M .

$$VA^2 = AM^2 + VM^2 = 32 + 144 = 176$$

$$VA = \sqrt{176} = 13.26 \dots$$

Answer: 13.3 cm

Tip: M is the centre of the base, so AM is half the diagonal, not half of 8 cm.

Marks: 1 mark for finding AM (or AC). 1 mark for $AM^2 + 12^2$. 1 mark for 13.3.

(b) Work out the angle between VA and the base.

M is directly below V , so the angle we want is angle VAM .

In triangle VMA : opposite = $VM = 12$, adjacent = $AM = 5.656 \dots$

$$\tan \theta = \frac{12}{5.656 \dots} = 2.121 \dots$$

$$\theta = \tan^{-1}(2.121 \dots) = 64.76 \dots$$

Answer: 64.8°

Marks: 1 mark for $\tan \theta = \frac{12}{AM}$ (or another correct ratio). 1 mark for 64.8.

4 (3 marks)

A cube has edges of length 5 cm.

A straight line is drawn inside the cube from one corner to the opposite corner.

Work out the angle between this line and the base of the cube.

The line goes from a bottom corner up to the opposite top corner.

Its “shadow” on the base is the diagonal of the base.

$$\text{Base diagonal} = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.071 \dots \text{ cm}$$

The height of the cube (5 cm) is opposite the angle and the base diagonal is adjacent.

$$\tan \theta = \frac{5}{7.071 \dots}$$

$$\theta = \tan^{-1}\left(\frac{5}{7.071\dots}\right) = 35.26\dots$$

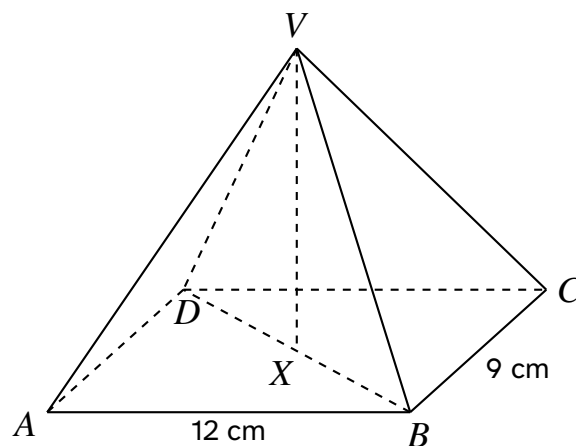
Answer: 35.3°

Tip: The angle is not 45°. The line rises 5 cm while it crosses the whole base diagonal, which is longer than 5 cm.

Marks: 1 mark for the base diagonal $\sqrt{50}$. 1 mark for $\tan \theta = \frac{5}{\sqrt{50}}$ (or another correct ratio). 1 mark for 35.3.

5 (3 marks)

Here is a pyramid.



Not drawn accurately

The base $ABCD$ of the pyramid is a horizontal rectangle.

The diagonals of the base cross at X , and the vertex V is directly above X .

$AB = 12$ cm and $BC = 9$ cm

The angle between VB and the base is 40°

Work out the height VX of the pyramid.

Base diagonal: $BD = \sqrt{12^2 + 9^2} = \sqrt{225} = 15$ cm

The diagonals of a rectangle cut each other in half, so X is the midpoint of BD :

$BX = 7.5$ cm

Triangle VBX has a right angle at X and angle $VBX = 40^\circ$.

VX is opposite the 40° angle and BX is adjacent.

$$VX = 7.5 \tan 40^\circ = 6.293 \dots$$

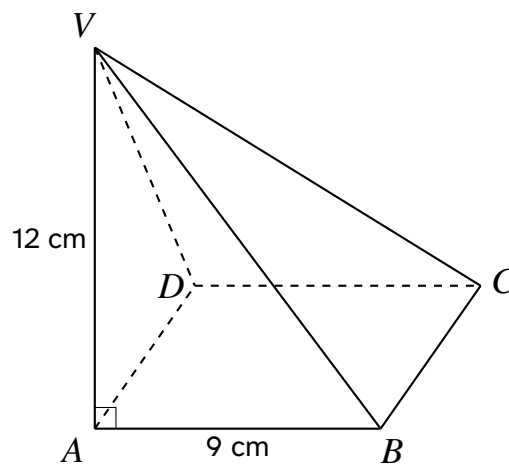
Answer: 6.29 cm

Tip: Use half of the diagonal BD . Neither 12 cm nor 9 cm is a side of the right-angled triangle VBX .

Marks: 1 mark for $BD = 15$ or $BX = 7.5$. 1 mark for $7.5 \tan 40^\circ$. 1 mark for 6.29.

6 (6 marks)

The base $ABCD$ of pyramid $VABCD$ is a horizontal square of side 9 cm.
The vertex V is directly above the corner A . $VA = 12$ cm



Not drawn accurately

- (a) Show that $VB = 15$ cm

V is vertically above A , so triangle VAB has a right angle at A .

$$VB^2 = VA^2 + AB^2 = 12^2 + 9^2 = 144 + 81 = 225$$

$$VB = \sqrt{225} = 15 \text{ cm } \checkmark$$

Marks: 1 mark for $12^2 + 9^2$. 1 mark for $\sqrt{225} = 15$.

- (b) Work out the angle between VC and the base $ABCD$.

A is directly below V , so the “shadow” of VC on the base is AC .

The angle we want is angle VCA .

Step 1: find the base diagonal AC .

$$AC = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.72 \dots \text{ cm}$$

Step 2: triangle VAC has a right angle at A .

Opposite = $VA = 12$, adjacent = $AC = 12.72 \dots$

$$\tan VCA = \frac{12}{12.72 \dots}$$

$$\text{Angle } VCA = \tan^{-1}\left(\frac{12}{12.72 \dots}\right) = 43.31 \dots$$

Answer: 43.3°

Tip: The angle is at C , between VC and AC . Draw triangle VAC on its own and mark the right angle at A before choosing the ratio.

Marks: 1 mark for $9^2 + 9^2$. 1 mark for $AC = 12.7$. 1 mark for $\tan^{-1}\left(\frac{12}{AC}\right)$. 1 mark for 43.3.
