

Venn Diagrams and Set Notation

Solutions

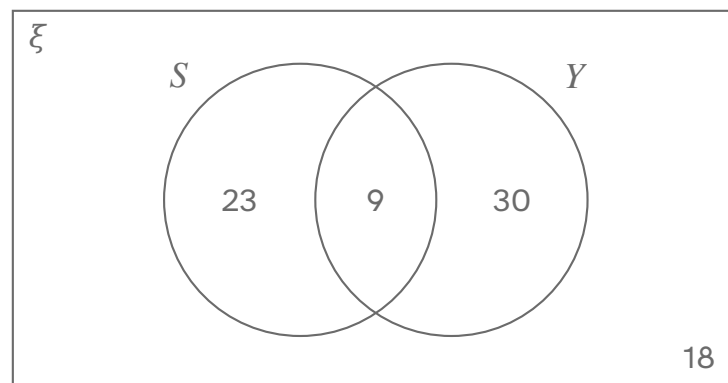
For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

The Venn diagram shows information about the 80 members of a gym.

S = members who swim Y = members who do yoga



(a) How many of the members do yoga?

Members who do yoga are everyone inside circle Y .

That includes the overlap: $9 + 30 = 39$

Answer: 39

Tip: Do not forget the 9 in the overlap. They swim **and** do yoga.

(b) How many of the members do **exactly one** of the two activities?

Exactly one activity means swimming only or yoga only.

$23 + 30 = 53$

Answer: 53

- (c) One of the members is chosen at random.
Write down the probability that the member does **not** swim.

Not swimming means outside circle S .

That is the yoga-only part and the part outside both circles.

$30 + 18 = 48$ members, out of 80.

Answer: $\frac{48}{80}$

Tip: Members outside both circles do not swim either. Include the 18.

2 (5 marks)

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

A = factors of 20

B = odd numbers

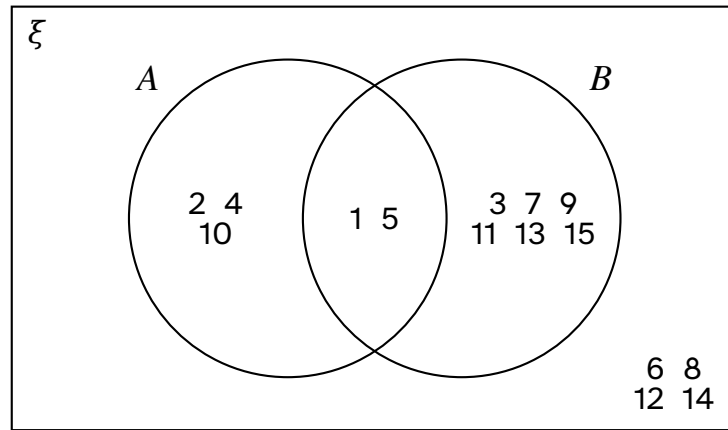
- (a) Complete the Venn diagram.

A (factors of 20): 1, 2, 4, 5, 10

B (odd numbers): 1, 3, 5, 7, 9, 11, 13, 15

In both: 1 and 5, so they go in the overlap.

In neither: 6, 8, 12, 14, so they go outside the circles.



Tip: 1 is a factor of every number, and it is odd, so it goes in the overlap. 20 is a factor of 20 but it is not in ξ .

Marks: 3 marks for a fully correct diagram. 2 marks for 3 of the 4 regions correct. 1 mark for 2 regions correct.

- (b) A number is chosen at random from ξ .
Write down the probability that the number is in $A \cap B$.

$A \cap B$ means in A **and** in B : the overlap.

The overlap has 2 numbers: 1 and 5

There are 15 numbers in ξ .

Answer: $\frac{2}{15}$

- (c) List the numbers in $A \cup B$.

$A \cup B$ means in A or B or both.

So list every number inside either circle, once each.

Answer: 1, 2, 3, 4, 5, 7, 9, 10, 11, 13, 15

Tip: $\cap$ (intersection) means **and**: in both sets. $\cup$ (union) means **or**: in either set, or both.

3 (5 marks)

There are 40 students in a club.

25 of the students play football.

18 of the students play tennis.

7 of the students play both football and tennis.

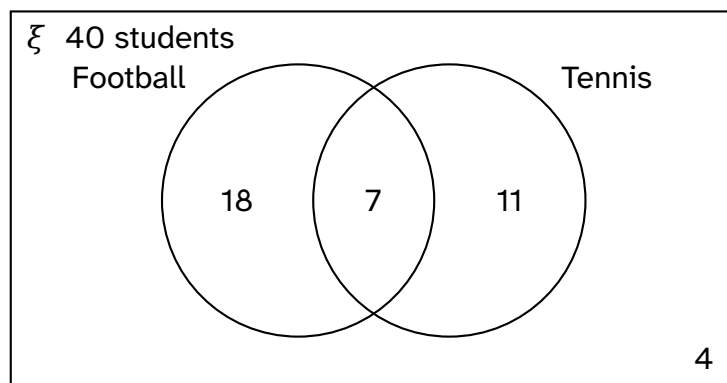
- (a) Complete the Venn diagram to show this information.

Start in the middle: 7 play both.

Football only: $25 - 7 = 18$

Tennis only: $18 - 7 = 11$

Neither: $40 - 18 - 7 - 11 = 4$



Answer: 18, 7, 11, 4

Tip: The 25 who play football includes the 7 who play both. So only $25 - 7 = 18$ go in the football-only part.

Marks: 1 mark for 7 in the overlap. 1 mark for 18 and 11. 1 mark for 4.

- (b) One of the students is picked at random.

Work out the probability that the student plays exactly one of the sports.

Exactly one sport means football only or tennis only.

$18 + 11 = 29$ students.

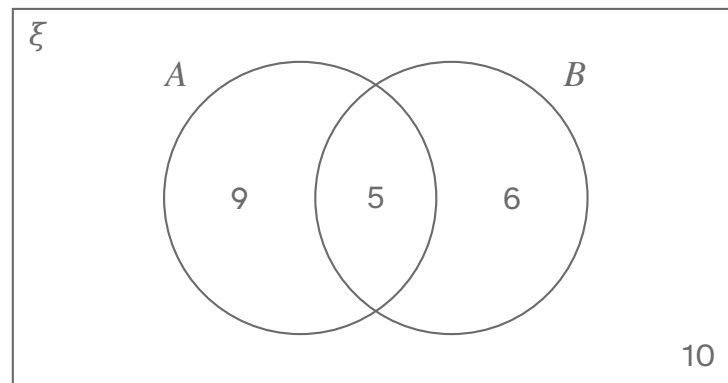
There are 40 students altogether.

Answer: $\frac{29}{40}$

Marks: 1 mark for $18 + 11$. 1 mark for $\frac{29}{40}$.

4 (2 marks)

The Venn diagram shows the number of elements in each region.



- (a) How many elements are in $A \cup B$?

$A \cup B$ is everything inside either circle.

$$9 + 5 + 6 = 20$$

Answer: 20

- (b) A member of ξ is chosen at random.

Write down $P(B')$, the probability that it is **not** in B .

Total in ξ : $9 + 5 + 6 + 10 = 30$

B' means **not** in B : the A -only part and the outside.

$$9 + 10 = 19$$

Answer: $\frac{19}{30}$

5 (2 marks)

There are 50 guests in a hotel.

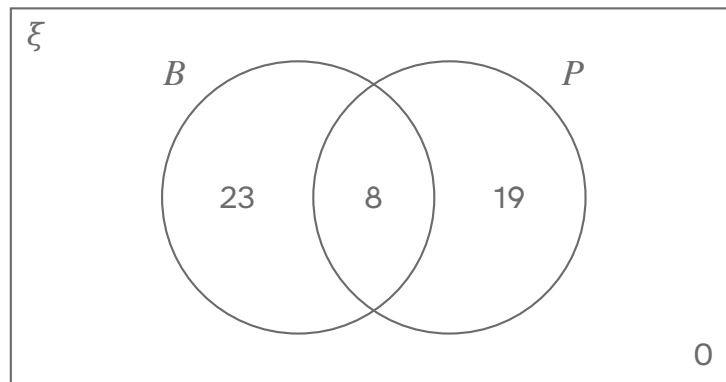
23 of the guests had breakfast in the hotel.

19 of the guests used the pool.

8 of the guests had breakfast and used the pool.

Leah draws this Venn diagram.

ξ = 50 guests B = had breakfast P = used the pool



- (a) Explain the mistake Leah has made.

The 23 who had breakfast includes the 8 who did both.

So breakfast only should be $23 - 8 = 15$, not 23.

In the same way, pool only should be $19 - 8 = 11$, not 19.

Answer: She used the totals 23 and 19 for the “only” regions. They include the 8 who did both, so they should be 15 and 11.

Tip: A number like “23 had breakfast” is the whole of circle B , including the overlap.

- (b) Work out how many of the guests did neither.

Correct regions: breakfast only 15, both 8, pool only 11

$15 + 8 + 11 = 34$ guests are inside the circles.

Neither: $50 - 34 = 16$

Answer: 16

6 (3 marks)

A cinema asked 120 customers what they bought.

40% of the customers bought popcorn.

38 customers bought a drink.

55 customers bought neither popcorn nor a drink.

$\xi = 120$ customers

P = customers who bought popcorn

D = customers who bought a drink

Complete this Venn diagram to represent the information.

Popcorn: 40% of 120 = $0.4 \times 120 = 48$

55 bought neither, so $120 - 55 = 65$ customers are inside the circles.

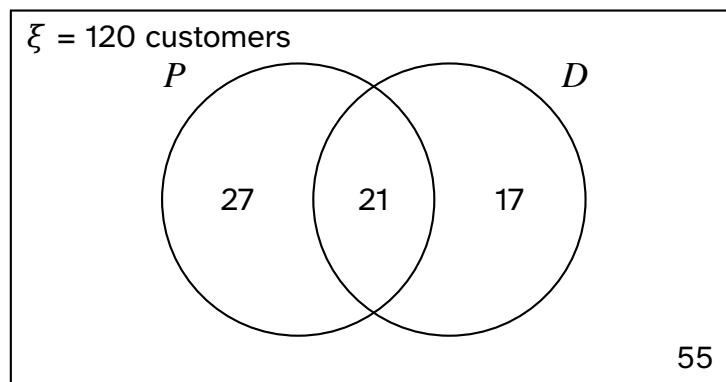
$48 + 38 = 86$, which is 21 more than 65.

So 21 customers were counted twice: they bought both.

Popcorn only: $48 - 21 = 27$

Drink only: $38 - 21 = 17$

Check: $27 + 21 + 17 + 55 = 120$ ✓



Answer: 27, 21, 17 and 55

Tip: 55 goes **outside** both circles. The circles only hold the other 65 customers, so the overlap is $48 + 38 - 65$.

Marks: 1 mark for 48 (popcorn). 1 mark for 21 in the overlap. 1 mark for a fully correct diagram.

7 (4 marks)

At a summer camp, every child did archery or canoeing or both.

45 children did archery only.

28 children did canoeing only.

One third of the children who did canoeing also did archery.

How many children were at the camp?

You can use the Venn diagram for your working.

Let x be the number of children who did both activities.

Children who did canoeing: $28 + x$

One third of them did archery too: $x = \frac{1}{3}(28 + x)$

Multiply both sides by 3: $3x = 28 + x$

$2x = 28$, so $x = 14$

Total: $45 + 14 + 28 = 87$

Check: canoeing $28 + 14 = 42$, and $\frac{1}{3}$ of 42 is 14 ✓

Answer: 87

Tip: The one third is of **all** the canoeists (including those who also did archery), not of the 28 who did canoeing only.

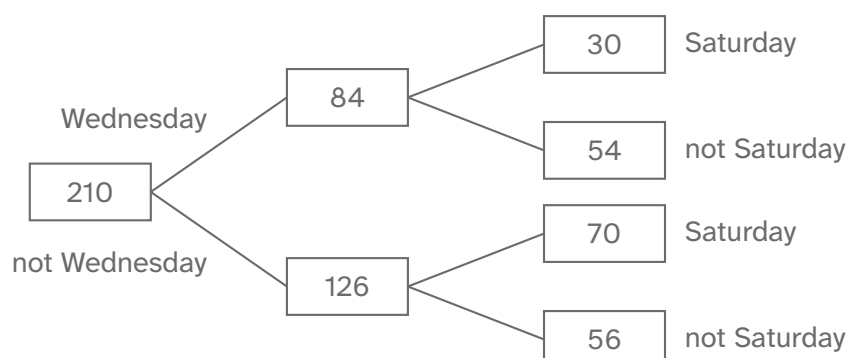
Marks: 1 mark for $28 + x$. 1 mark for $x = \frac{1}{3}(28 + x)$ or $3x = 28 + x$. 1 mark for $x = 14$. 1 mark for 87.

8 (4 marks)

A walking group has 210 members.

One week, the group organised a walk on Wednesday and a walk on Saturday.

The frequency tree shows how many members went on the walks.



$\xi = 210$ members, W = went on the Wednesday walk, S = went on the Saturday walk

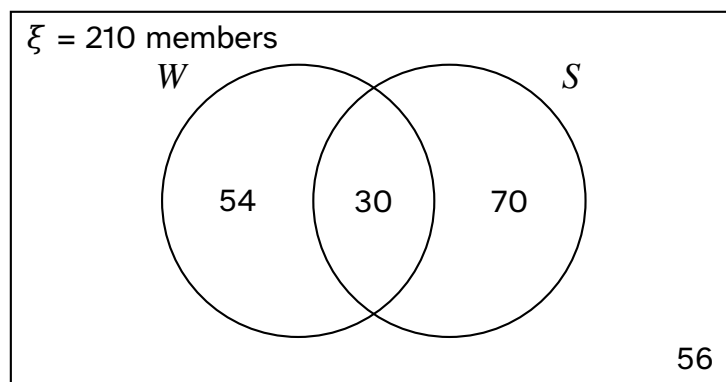
Show the information on the Venn diagram.

Wednesday **and** Saturday: 30, so this goes in the overlap.

Wednesday, not Saturday: 54, so this goes in W only.

Not Wednesday, Saturday: 70, so this goes in S only.

Neither: 56, so this goes outside the circles.



Answer: 54, 30, 70 and 56

Tip: Work out what each end box of the tree means before you place it. Do not copy the numbers across in the same order.

Marks: 1 mark for each of 30, 54, 70 and 56 in the correct region.