

Vectors

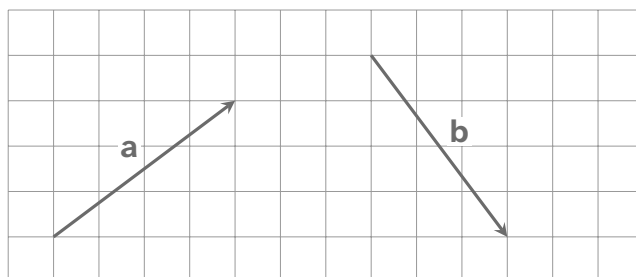
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Vectors **a** and **b** are drawn on the grid.



(a) Write **a** as a column vector.

Count squares from the start of the arrow to the end.

4 squares right, so the top number is 4.

3 squares up, so the bottom number is 3.

Answer: $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(b) Write **b** as a column vector.

3 squares right, so the top number is 3.

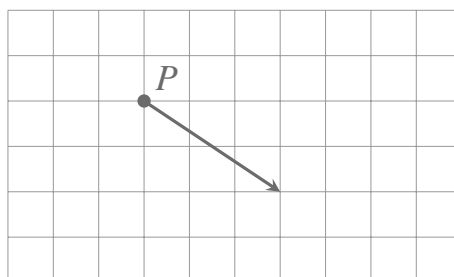
4 squares down, so the bottom number is -4 .

Answer: $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

Tip: Follow the direction of the arrow. Down and left are negative.

2 (1 mark)

On the grid, draw the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ starting at point P .



Top number 3: go 3 squares right.

Bottom number -2 : go 2 squares down.

Draw a straight line from P to that point, with an arrowhead at the end.

Answer: Arrow drawn

3 (2 marks)

A is the point $(1, 4)$ and B is the point $(6, 2)$.

(a) Write down the column vector $\overrightarrow{AB}$

$\overrightarrow{AB}$ goes from A to B : end minus start.

Across: $6 - 1 = 5$

Up: $2 - 4 = -2$

Answer: $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(b) Write down the column vector $\overrightarrow{BA}$

$\overrightarrow{BA}$ goes the opposite way to $\overrightarrow{AB}$.

So $\overrightarrow{BA} = -\overrightarrow{AB}$: change the sign of both numbers.

Answer: $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

Tip: Reversing a vector changes both signs. Do not swap the top and bottom numbers.

4 (5 marks)

$\mathbf{p} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Work out, as a column vector,

(a) $\mathbf{p} + \mathbf{q}$

Add the top numbers: $2 + 4 = 6$

Add the bottom numbers: $5 + (-1) = 4$

Answer: $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Tip: A column vector is not a fraction. Work on the top numbers and the bottom numbers separately, and never draw a fraction line.

(b) $\mathbf{p} - \mathbf{q}$

Subtract the top numbers: $2 - 4 = -2$

Subtract the bottom numbers: $5 - (-1) = 5 + 1 = 6$

Answer: $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$

(c) $3\mathbf{p}$

Multiply both numbers by 3.

$$3 \times 2 = 6$$

$$3 \times 5 = 15$$

Answer: $\begin{pmatrix} 6 \\ 15 \end{pmatrix}$

(d) $2\mathbf{p} - 3\mathbf{q}$

$$2\mathbf{p} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$$

$$3\mathbf{q} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$$

Top: $4 - 12 = -8$

Bottom: $10 - (-3) = 10 + 3 = 13$

Answer: $\begin{pmatrix} -8 \\ 13 \end{pmatrix}$

Tip: Multiply **both** numbers in each vector first. Then take care with the minus signs:

$$10 - (-3) = 13.$$

Marks: 1 mark for $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ or $\begin{pmatrix} 12 \\ -3 \end{pmatrix}$. 1 mark for the correct answer.

5 (1 mark)

Circle the vector that is equal to $-2 \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Options: $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Multiply both numbers by -2 .

$$-2 \times 3 = -6$$

$$-2 \times (-1) = 2$$

Answer: $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$

6 (1 mark)

$$\mathbf{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Circle the vector $\mathbf{a} - \mathbf{b}$

Options: $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

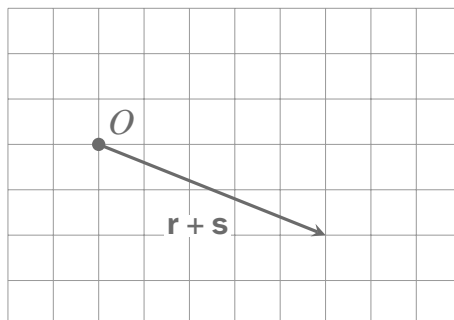
$$\text{Top: } 6 - 2 = 4$$

$$\text{Bottom: } 1 - 4 = -3$$

Answer: $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

7 (2 marks)

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } \mathbf{s} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$



(a) On the grid, draw the vector $\mathbf{r} + \mathbf{s}$ starting at point O .

$$\mathbf{r} + \mathbf{s} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

So draw an arrow from O going 5 right and 2 down.

Answer: Arrow $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(b) Write $\mathbf{r} + \mathbf{s}$ as a column vector.

$$\text{Top: } 3 + 2 = 5$$

$$\text{Bottom: } 1 + (-3) = -2$$

Answer: $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

8 (2 marks)

$$\mathbf{m} + \mathbf{n} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Work out $\mathbf{n}$ as a column vector.

$$\mathbf{n} = (\mathbf{m} + \mathbf{n}) - \mathbf{m}$$

$$\mathbf{n} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Top: $7 - 2 = 5$

Bottom: $1 - 5 = -4$

Answer: $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

Marks: 1 mark for subtracting the vectors. 1 mark for the correct answer.

9 (2 marks)

$$2\mathbf{a} + \mathbf{b} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} \text{ and } \mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Work out $\mathbf{b}$ as a column vector.

First work out $2\mathbf{a}$: $2\mathbf{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

$$\mathbf{b} = (2\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$$

$$\mathbf{b} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

Top: $7 - 6 = 1$

Bottom: $4 - (-2) = 6$

Answer: $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$

Marks: 1 mark for $2\mathbf{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ or a correct subtraction. 1 mark for the correct answer.

10 (2 marks)

R is the point $(4, -1)$.

$$\overrightarrow{RS} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$$

Work out the coordinates of S .

Start at R and move by the vector $\overrightarrow{RS}$.

$$x: 4 + (-6) = -2$$

$$y: -1 + 3 = 2$$

Answer: $(-2, 2)$

Marks: 1 mark for a correct method, e.g. $(4 - 6, -1 + 3)$. 1 mark for the correct answer.

11 (3 marks)

P is the point $(1, 2)$.

$$\overrightarrow{PQ} = 2\mathbf{a} - \mathbf{b} \text{ where } \mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

Work out the coordinates of Q .

$$2\mathbf{a} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$\text{Top: } 6 - (-1) = 7$$

$$\text{Bottom: } 2 - 4 = -2$$

$$\text{So } \overrightarrow{PQ} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

Start at $P(1, 2)$ and move 7 right and 2 down:

$$Q = (1 + 7, 2 - 2) = (8, 0)$$

Answer: (8, 0)

Tip: Work out the vector first, then add it to the coordinates of P . $\overrightarrow{PQ}$ is a movement, not the position of Q .

Marks: 1 mark for $2\mathbf{a} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$. 1 mark for $\begin{pmatrix} 7 \\ -2 \end{pmatrix}$. 1 mark for adding your vector to (1, 2) correctly.
