

Spheres, Cones and Pyramids

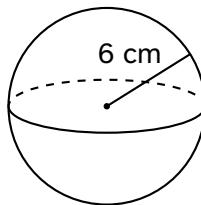
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A sphere has radius 6 cm.



Not drawn accurately

Work out the volume of the sphere.

Give your answer to 1 decimal place.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

Here $r = 6$.

$$V = \frac{4}{3} \times \pi \times 6^3$$

$$= \frac{4}{3} \times \pi \times 216 = 904.77 \dots$$

Round to 1 decimal place: 904.8

Answer: 904.8 cm³

Tip: The radius is **cubed** ($6^3 = 216$), not squared and not multiplied by 3.

Marks: 1 mark for $\frac{4}{3} \times \pi \times 6^3$. 1 mark for 904.8.

2 (2 marks)

A hemisphere has radius 5 cm.

Work out the volume of the hemisphere.

Give your answer to 1 decimal place.

A hemisphere is half of a sphere.

$$\text{Volume of the whole sphere} = \frac{4}{3} \times \pi \times 5^3 = 523.59 \dots$$

$$\text{Volume of the hemisphere} = 523.59 \dots \div 2 = 261.79 \dots$$

Round to 1 decimal place: 261.8

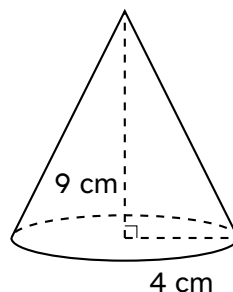
Answer: 261.8 cm³

Tip: Remember to halve. Many students find the volume of the whole sphere and stop there.

Marks: 1 mark for $\frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$. 1 mark for 261.8.

3 (2 marks)

A cone has radius 4 cm and perpendicular height 9 cm.



Not drawn accurately

Work out the volume of the cone.

Give your answer in terms of π .

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

Here $r = 4$ and $h = 9$.

$$V = \frac{1}{3} \times \pi \times 4^2 \times 9$$

$$= \frac{1}{3} \times 16 \times 9 \times \pi$$

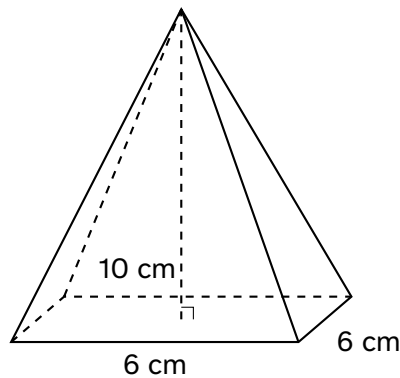
$$= 48\pi$$

Answer: $48\pi \text{ cm}^3$

Marks: 1 mark for $\frac{1}{3} \times \pi \times 4^2 \times 9$. 1 mark for 48π .

4 (2 marks)

A pyramid has a square base of side 6 cm.
Its perpendicular height is 10 cm.



Not drawn accurately

Work out the volume of the pyramid.

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$\text{Area of the square base} = 6 \times 6 = 36 \text{ cm}^2$$

$$V = \frac{1}{3} \times 36 \times 10$$

$$= 120$$

Answer: 120 cm^3

Tip: Do not forget the $\frac{1}{3}$. $6 \times 6 \times 10 = 360$ is the volume of a cuboid.

Marks: 1 mark for $\frac{1}{3} \times 6 \times 6 \times 10$. 1 mark for 120.

5 (2 marks)

A ball is a sphere with radius 2.5 cm.
Work out the surface area of the ball.
Give your answer to 1 decimal place.

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$= 4 \times \pi \times 2.5^2$$

$$= 4 \times \pi \times 6.25 = 78.53 \dots$$

Round to 1 decimal place: 78.5

Answer: 78.5 cm²

Marks: 1 mark for $4 \times \pi \times 2.5^2$. 1 mark for 78.5.

6 (2 marks)

A cone has radius 3 cm and slant height 7 cm.
Work out the curved surface area of the cone.
Give your answer to 1 decimal place.

Curved surface area of a cone = πrl , where l is the slant height.

$$= \pi \times 3 \times 7$$

$$= 21\pi = 65.97 \dots$$

Round to 1 decimal place: 66.0

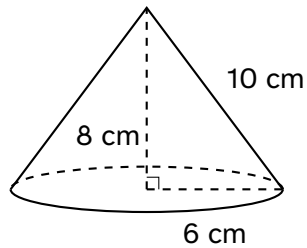
Answer: 66.0 cm²

Marks: 1 mark for $\pi \times 3 \times 7$. 1 mark for 66.0 (or 65.97...).

7 (2 marks)

Here is a cone.

The radius is 6 cm, the perpendicular height is 8 cm and the slant height is 10 cm.



Not drawn accurately

- (a) Beth tries to work out the curved surface area of the cone in terms of π

$$\text{Curved surface area of the cone} = \pi \times 6 \times 8 = 48\pi \text{ cm}^2$$

What mistake has she made?

The formula is πrl , where l is the **slant** height.

Beth used 8 cm, which is the perpendicular height.

She should have used the slant height, 10 cm.

Answer: She used the perpendicular height instead of the slant height

- (b) Work out the correct curved surface area of the cone.

Give your answer in terms of π

$$\text{Curved surface area} = \pi rl$$

$$= \pi \times 6 \times 10$$

$$= 60\pi$$

Answer: $60\pi \text{ cm}^2$

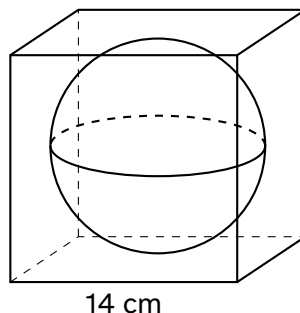
8 (4 marks)

A glass ball is a sphere with radius 7 cm.

It is packed in a box in the shape of a cube with edges of length 14 cm.

The ball touches every face of the box.

The rest of the space inside the box is filled with foam.



Not drawn accurately

Work out the percentage of the space inside the box that is filled with foam.

Give your answer to 1 decimal place.

$$\text{Volume of the box} = 14 \times 14 \times 14 = 2744 \text{ cm}^3$$

$$\text{Volume of the ball} = \frac{4}{3} \times \pi \times 7^3$$

$$= \frac{4}{3} \times \pi \times 343 = 1436.75 \dots \text{ cm}^3$$

$$\text{Volume of the foam} = 2744 - 1436.75 \dots = 1307.24 \dots \text{ cm}^3$$

$$\text{Percentage of the box filled with foam} = \frac{1307.24 \dots}{2744} \times 100 = 47.64 \dots$$

To 1 decimal place: 47.6%

Answer: 47.6%

Tip: Use the radius (7 cm) in the sphere formula, not the width of the box (14 cm). Then divide by the volume of the **box**: 52.4% is the part filled by the ball, not the foam.

Marks: 1 mark for 14^3 (2744). 1 mark for $\frac{4}{3} \times \pi \times 7^3$ (1436.7...). 1 mark for $\frac{2744 - 1436.7 \dots}{2744} \times 100$. 1 mark for 47.6.

9 (4 marks)

A shop sells two chocolates.

Chocolate A is a square-based pyramid. Its base has side 3 cm and its perpendicular height is 4 cm.

Chocolate B is a cone. Its radius is 1.8 cm and its perpendicular height is 3.6 cm.

Which chocolate has the greater volume?

You must show your working.

Chocolate A (pyramid): area of base = $3 \times 3 = 9 \text{ cm}^2$

$$\text{Volume of A} = \frac{1}{3} \times 9 \times 4 = 12 \text{ cm}^3$$

$$\text{Chocolate B (cone): } \frac{1}{3} \times \pi \times 1.8^2 \times 3.6$$

$$\text{Volume of B} = 12.21 \dots \text{ cm}^3$$

12.21 ... is bigger than 12.

Answer: Chocolate B (12.2 cm^3 compared with 12 cm^3)

Tip: Do not forget the $\frac{1}{3}$ in **both** formulas. The volumes are close, so do not round too early. Label your working so it is clear which volume is which.

Marks: 1 mark for $\frac{1}{3} \times 3 \times 3 \times 4$. 1 mark for $\frac{1}{3} \times \pi \times 1.8^2 \times 3.6$. 1 mark for both volumes correct (12 and 12.2...). 1 mark for B, with both volumes correct.

10 (4 marks)

A water cooler holds 18 litres of water.

Each paper cup is a cone with radius 3 cm and perpendicular height 9 cm.

$$1 \text{ litre} = 1000 \text{ cm}^3$$

How many cups can be filled completely from the full water cooler?

You must show your working.

$$\text{Volume of one cup} = \frac{1}{3} \times \pi \times 3^2 \times 9 = 27\pi = 84.82 \dots \text{ cm}^3$$

$$\text{Water in the cooler: } 18 \text{ litres} = 18 \times 1000 = 18\,000 \text{ cm}^3$$

$$\text{Number of cups: } 18\,000 \div 84.82 \dots = 212.2 \dots$$

Only whole cups count, so round down.

Answer: 212 cups

Tip: Change litres to cm^3 ($\times 1000$) before you divide. The last cup is not full, so round **down**.

Marks: 1 mark for $\frac{1}{3} \times \pi \times 3^2 \times 9$. 1 mark for 84.8 (or 27π). 1 mark for $18\,000 \div$ your volume. 1 mark for 212.
