

Special Sequences

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

- (a) Write down the first five square numbers.

Square numbers are 1×1 , 2×2 , 3×3 , ...

$$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25$$

Answer: 1, 4, 9, 16, 25

- (b) Write down the first five triangular numbers.

Start at 1, then add 2, then 3, then 4, then 5.

$$1, 1 + 2 = 3, 3 + 3 = 6, 6 + 4 = 10, 10 + 5 = 15$$

Answer: 1, 3, 6, 10, 15

- (c) Write down the first four cube numbers.

Cube numbers are $1 \times 1 \times 1$, $2 \times 2 \times 2$, ...

$$1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64$$

Answer: 1, 8, 27, 64

2 (3 marks)

- (a) Circle the geometric progression.

Options: 3, 6, 9, 12, 3, 6, 12, 24, 3, 6, 10, 15, 3, 6, 9, 15

In a geometric progression you multiply by the same number each time.

$$3 \times 2 = 6, 6 \times 2 = 12, 12 \times 2 = 24 \quad \checkmark$$

3, 6, 9, 12 adds 3 each time (linear). 3, 6, 10, 15 are triangular numbers. 3, 6, 9, 15 is Fibonacci-type.

Answer: 3, 6, 12, 24

Tip: Geometric means you **multiply** by the same number each time. Linear means you **add** the same number each time.

- (b) Circle the Fibonacci-type sequence.

Options: 2, 5, 8, 11, 2, 5, 10, 17, 2, 5, 7, 12, 2, 5, 10, 20

In a Fibonacci-type sequence each term is the sum of the two terms before it.

$$2 + 5 = 7 \quad \text{and} \quad 5 + 7 = 12 \quad \checkmark$$

Answer: 2, 5, 7, 12

- (c) Circle the quadratic sequence.

Options: 1, 4, 7, 10, 2, 6, 18, 54, 3, 5, 8, 13, 2, 5, 10, 17

In a quadratic sequence the differences change by the same amount each time.

2, 5, 10, 17: the differences are 3, 5, 7, which go up by 2 each time $\checkmark$

(1, 4, 7, 10 is linear, 2, 6, 18, 54 is geometric and 3, 5, 8, 13 is Fibonacci-type.)

Answer: 2, 5, 10, 17

Tip: Look at the differences. Equal differences mean linear. Differences that go up by the same amount each time mean quadratic.

3 (3 marks)

- (a) Write the missing term in the geometric progression.

5 20 _____ 320

$20 \div 5 = 4$, so you multiply by 4 each time.

$$20 \times 4 = 80$$

Check: $80 \times 4 = 320$ ✓

Answer: 80

- (b) A geometric progression starts 4 16
Work out the 5th term.

$16 \div 4 = 4$, so the multiplier is 4.

3rd term: $16 \times 4 = 64$

4th term: $64 \times 4 = 256$

5th term: $256 \times 4 = 1024$

Answer: 1024

Tip: In a geometric progression you multiply each time. Adding 12 each time (giving 28, 40, ...) treats it as a linear sequence, which is wrong.

Marks: 1 mark for using $\times 4$. 1 mark for 1024.

4 (2 marks)

In a Fibonacci-type sequence, each term after the 2nd term is the sum of the two terms before it.

The 1st term is 7 and the 2nd term is -12

Work out the 5th term.

Add the two previous terms each time.

3rd term: $7 + (-12) = 7 - 12 = -5$

4th term: $-12 + (-5) = -12 - 5 = -17$

5th term: $-5 + (-17) = -5 - 17 = -22$

Answer: -22

Tip: Adding a negative number is the same as subtracting. Take care with the signs:

$7 + (-12) = -5$, not 5 or 19.

Marks: 1 mark for -5 and -17 as the 3rd and 4th terms. 1 mark for -22 .

5 (3 marks)

A Fibonacci-type sequence is made by adding the previous two terms.

The 2nd term is 9 and the 4th term is 23

Work out the 1st term.

Each term is the sum of the two terms before it.

So 2nd term + 3rd term = 4th term.

$9 + 3\text{rd term} = 23$, so the 3rd term is $23 - 9 = 14$.

Also 1st term + 2nd term = 3rd term.

$1\text{st term} + 9 = 14$, so the 1st term is $14 - 9 = 5$.

Check: 5, 9, 14, 23 ✓

Answer: 5

Tip: Work backwards one term at a time. Find the 3rd term first, then use it to find the 1st term.

Marks: 1 mark for a correct method to find the 3rd term (e.g. $23 - 9$). 1 mark for 14. 1 mark for 5.

6 (1 mark)

Maya says,

‘50 is a triangular number.’

Is Maya correct? Tick a box.

Give a reason for your answer.

The triangular numbers are

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

45 and 55 are next to each other, so 50 is not a triangular number.

Answer: No, the triangular numbers go 45, 55

7 (2 marks)

The 1st term of a geometric progression is 5

The 3rd term is 45

Every term of the progression is positive.

Work out the 2nd term.

Let the multiplier be r .

To go from the 1st term to the 3rd term you multiply by r twice.

$$5 \times r \times r = 45$$

$$\text{So } r^2 = 45 \div 5 = 9$$

$$r = 3 \text{ (because } 3 \times 3 = 9, \text{ and the terms are positive)}$$

$$\text{2nd term: } 5 \times 3 = 15$$

Check: 5, 15, 45 is correct.

Answer: 15

Tip: Do not just find the number halfway between 5 and 45 (that would be 25). A geometric progression multiplies each time, so find the multiplier first.

Marks: 1 mark for $r^2 = 9$ (or $r = 3$). 1 mark for 15.

8 (3 marks)

Work out the next term in each sequence.

(i) 1 8 27 64 ...

(ii) 2 3 5 8 13 ...

(iii) 100 50 25 12.5 ...

(i) These are the cube numbers $1^3, 2^3, 3^3, 4^3$.

$$\text{Next: } 5^3 = 5 \times 5 \times 5 = 125$$

(ii) Each term is the sum of the two before (Fibonacci-type).

Next: $8 + 13 = 21$

(iii) Each term is half of the one before.

Next: $12.5 \div 2 = 6.25$

Answer: 125, 21, 6.25

Marks: 1 mark for each correct answer.

9 (2 marks)

Here are the first four terms of a quadratic sequence.

40 38 34 28 ...

Work out the 6th term.

Find the differences between the terms:

First differences: $-2, -4, -6$

These go down by 2 each time (the second difference is -2).

So the next differences are -8 and then -10 .

5th term: $28 - 8 = 20$

6th term: $20 - 10 = 10$

Answer: 10

Tip: The question asks for the 6th term, not the next term. Label each term with its position so you stop at the right place.

Marks: 1 mark for the differences $-2, -4, -6$ or the next difference -8 . 1 mark for 10.

10 (2 marks)

The first two terms of a Fibonacci-type sequence are p and q
Each term after that is the sum of the two terms before it.

Show that the 5th term minus the 2nd term is always equal to twice the 3rd term.

1st term: p

2nd term: q

3rd term: $p + q$

4th term: $q + (p + q) = p + 2q$

5th term: $(p + q) + (p + 2q) = 2p + 3q$

5th term – 2nd term: $2p + 3q - q = 2p + 2q$

Twice the 3rd term: $2(p + q) = 2p + 2q$

These are the same, so the 5th term minus the 2nd term is twice the 3rd term.

Tip: Write every term in terms of p and q first. Trying one set of numbers does not show that it is **always** true.

Marks: 1 mark for the 5th term, $2p + 3q$. 1 mark for showing that $2p + 3q - q = 2(p + q)$.

11 (4 marks)

The first four terms of an arithmetic progression, P , are

8 14 20 26

The first four terms of a geometric progression, Q , are

4 8 16 32

Show that 128 is a term of both sequences.

Q multiplies by 2 each time.

4, 8, 16, 32, 64, 128

So 128 is the 6th term of Q .

P goes up by 6 each time, so its n th term starts $6n$.

When $n = 1$: $6 \times 1 = 6$, but the first term is 8, so add 2.

The n th term of P is $6n + 2$.

Solve $6n + 2 = 128$:

Subtract 2: $6n = 126$

Divide by 6: $n = 21$

21 is a whole number, so 128 is the 21st term of P .

So 128 is a term of both sequences.

Answer: 128 is the 6th term of Q and the 21st term of P .

Tip: To show a number is in a linear sequence, make an equation with the n th term. The number is a term only if n is a whole number.

Marks: 1 mark for showing 128 is in Q . 1 mark for $6n + 2$. 1 mark for $6n + 2 = 128$ (or $128 - 2 = 126$ and $126 \div 6$). 1 mark for $n = 21$ with a conclusion.
