

Solving Quadratics by Factorising

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Write down the two solutions of $(x + 3)(x - 5) = 0$

Two things multiply to give 0, so one of them must be 0.

$$x + 3 = 0 \text{ gives } x = -3$$

$$x - 5 = 0 \text{ gives } x = 5$$

Answer: $x = -3$ and $x = 5$

Tip: The solutions have the **opposite** signs to the numbers in the brackets. $(x + 3)$ gives $x = -3$, not $x = 3$.

2 (1 mark)

Which of these is a solution of $x^2 - 4x - 12 = 0$?

Circle your answer.

Options: -6, -2, 2, 4

Factorise: two numbers that multiply to -12 and add to -4 are -6 and +2.

$$(x - 6)(x + 2) = 0$$

$$\text{So } x = 6 \text{ or } x = -2$$

Only -2 is in the list.

Check: $(-2)^2 - 4 \times (-2) - 12 = 4 + 8 - 12 = 0$ ✓

Answer: -2

Tip: You can also test each value by substituting it into the equation. The one that gives 0 is a solution.

3 (2 marks)

Factorise and solve $x^2 - 9x + 14 = 0$

Find two numbers that multiply to +14 and add to -9 .

They are -2 and -7 .

So $(x - 2)(x - 7) = 0$

$x - 2 = 0$ gives $x = 2$

$x - 7 = 0$ gives $x = 7$

Answer: $x = 2$ and $x = 7$

Marks: 1 mark for $(x - 2)(x - 7)$. 1 mark for both solutions.

4 (3 marks)

Solve $x^2 + 6x + 8 = 0$

Find two numbers that multiply to +8 and add to +6.

They are +2 and +4.

So $(x + 2)(x + 4) = 0$

$x + 2 = 0$ gives $x = -2$

$x + 4 = 0$ gives $x = -4$

Answer: $x = -2$ and $x = -4$

Marks: 1 mark for $(x + 2)(x + 4)$. 1 mark for one correct solution. 1 mark for both.

5 (3 marks)

Solve $x^2 - 2x - 24 = 0$

Find two numbers that multiply to -24 and add to -2 .

They are -6 and $+4$.

So $(x - 6)(x + 4) = 0$

$x - 6 = 0$ gives $x = 6$

$x + 4 = 0$ gives $x = -4$

Answer: $x = 6$ and $x = -4$

Tip: Trying numbers might find $x = 6$, but you would probably miss $x = -4$. Factorising gives you both.

Marks: 1 mark for $(x - 6)(x + 4)$. 1 mark for one correct solution. 1 mark for both.

6 (3 marks)

Solve $x^2 + 2x - 15 = 0$

Find two numbers that multiply to -15 and add to $+2$.

They are $+5$ and -3 .

So $(x + 5)(x - 3) = 0$

$x + 5 = 0$ gives $x = -5$

$x - 3 = 0$ gives $x = 3$

Answer: $x = -5$ and $x = 3$

Marks: 1 mark for $(x + 5)(x - 3)$. 1 mark for one correct solution. 1 mark for both.

7 (3 marks)

(a) Solve $x^2 = 169$

Square root both sides.

$$\sqrt{169} = 13$$

But $(-13)^2 = 169$ as well.

So $x = 13$ or $x = -13$.

Answer: $x = 13$ and $x = -13$

Marks: 1 mark for 13. 1 mark for -13.

(b) Ella says,

‘The solutions of $x^2 + 25 = 0$ are $x = 5$ and $x = -5$ ’

Explain why Ella is wrong.

Substitute $x = 5$: $5^2 + 25 = 25 + 25 = 50$, not 0.

Substitute $x = -5$: $(-5)^2 + 25 = 25 + 25 = 50$, not 0.

In fact, any number squared is 0 or positive, so $x^2 + 25$ is always at least 25.

So $x^2 + 25 = 0$ has no solutions.

Answer: $5^2 + 25 = 50$ and $(-5)^2 + 25 = 50$, not 0 (a square is never negative, so there are no solutions).

Tip: Ella has solved $x^2 = 25$ by mistake. Moving +25 to the other side gives $x^2 = -25$, and no number squared is negative.

8 (3 marks)

Solve $x^2 - 9x = 0$

Both terms contain x , so take x out as a common factor.

$$x(x - 9) = 0$$

So $x = 0$ or $x - 9 = 0$

$$x = 0 \text{ or } x = 9$$

Answer: $x = 0$ and $x = 9$

Tip: Do not divide both sides by x . You would lose the solution $x = 0$.

Marks: 1 mark for $x(x - 9)$. 1 mark for one correct solution. 1 mark for both.

9 (2 marks)

Solve $x^2 - 64 = 0$

This is a difference of two squares: $x^2 - 8^2$.

So $(x + 8)(x - 8) = 0$

$$x = -8 \text{ or } x = 8$$

(Or: $x^2 = 64$, so $x = 8$ or $x = -8$.)

Answer: $x = 8$ and $x = -8$

Marks: 1 mark for $(x + 8)(x - 8)$ or $x^2 = 64$. 1 mark for both solutions.

10 (2 marks)

Solve $x^2 - 8x + 16 = 0$

Find two numbers that multiply to +16 and add to -8 .

They are -4 and -4 .

So $(x - 4)(x - 4) = 0$, which is $(x - 4)^2 = 0$

Both brackets give $x = 4$.

There is only one solution (a repeated root).

Answer: $x = 4$

Marks: 1 mark for $(x - 4)^2$. 1 mark for 4.

11 (2 marks)

Liam says,

‘The solutions of $(x - 3)(x + 7) = 0$ are $x = -3$ and $x = 7$ ’

Is Liam correct? Tick a box.

Give a reason for your answer.

$$x - 3 = 0 \text{ gives } x = 3$$

$$x + 7 = 0 \text{ gives } x = -7$$

Liam has the signs the wrong way round.

Check: $(-3 - 3)(-3 + 7) = -6 \times 4 = -24$, not 0.

Answer: No. The solutions are $x = 3$ and $x = -7$.

Marks: 1 mark for No. 1 mark for the correct solutions, 3 and -7 .

12 (4 marks)

A rectangle has length $(x + 3)$ cm and width x cm.

The area of the rectangle is 40 cm^2 .

Work out the value of x .

Area of a rectangle = length $\times$ width

$$\text{So } x(x + 3) = 40$$

$$\text{Expand the bracket: } x^2 + 3x = 40$$

$$\text{Subtract 40 from both sides: } x^2 + 3x - 40 = 0$$

Two numbers that multiply to -40 and add to $+3$: $+8$ and -5

$$\text{So } (x + 8)(x - 5) = 0$$

$$x = -8 \text{ or } x = 5$$

A length cannot be negative, so $x = 5$.

Check: $5 \times 8 = 40$ ✓

Answer: $x = 5$

Tip: Use the area (length $\times$ width), not the perimeter. At the end, reject $x = -8$, because a length cannot be negative.

Marks: 1 mark for $x(x + 3) = 40$. 1 mark for $x^2 + 3x - 40 = 0$. 1 mark for $(x + 8)(x - 5)$. 1 mark for 5 only.

13 (3 marks)

Solve $x^2 + 3x = 10$

First rearrange so that one side is 0.

Subtract 10 from both sides: $x^2 + 3x - 10 = 0$

Two numbers that multiply to -10 and add to $+3$: $+5$ and -2

So $(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

Answer: $x = -5$ and $x = 2$

Tip: You can only use the “one bracket must be 0” rule when the other side is 0. Rearrange first.

Marks: 1 mark for $x^2 + 3x - 10 = 0$. 1 mark for $(x + 5)(x - 2)$. 1 mark for both solutions.
