

# Solving Equations Using Graphs

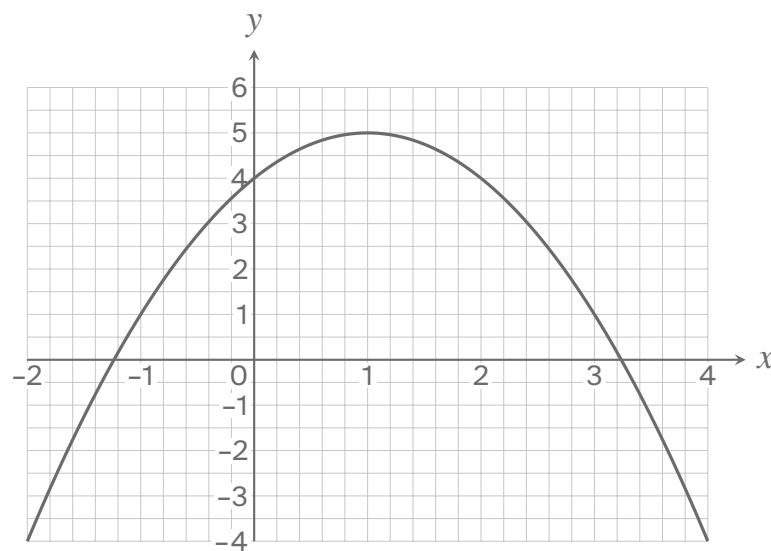
## Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (7 marks)

Here is the graph of  $y = 4 + 2x - x^2$



(a) Use the graph to estimate the solutions of  $4 + 2x - x^2 = 0$

The solutions of  $4 + 2x - x^2 = 0$  are where  $y = 0$ .

That is where the curve crosses the  $x$ -axis.

Read off the two  $x$ -values:  $x \approx -1.2$  and  $x \approx 3.2$

Answers within 0.1 of these are fine.

**Answer:**  $x \approx -1.2$  and  $x \approx 3.2$

**Tip:** Give the solutions as  $x$ -values, not coordinates. The  $y$ -intercept (4) is **not** a solution.

*Marks:* 1 mark for each solution.

- (b) Write down the coordinates of the turning point of the curve.

The turning point is the highest point of the curve, at  $x = 1$ .

Work out  $y$  exactly:  $4 + 2 \times 1 - 1^2 = 4 + 2 - 1 = 5$

**Answer:** (1, 5)

- (c) Use the graph to solve  $4 + 2x - x^2 = 1$

Draw a horizontal line across the grid at  $y = 1$ .

It meets the curve twice. Read down to the  $x$ -axis:

$x = -1$  and  $x = 3$

Check  $x = 3$ :  $4 + 6 - 9 = 1$  ✓

**Answer:**  $x = -1$  and  $x = 3$

*Marks:* 1 mark for using the line  $y = 1$ . 1 mark for both solutions.

- (d) Use the graph to estimate the solutions of  $4 + 2x - x^2 = 2$

Draw a horizontal line at  $y = 2$ .

Read down from where it meets the curve:

$x \approx -0.7$  and  $x \approx 2.7$

Answers within 0.1 of these are fine.

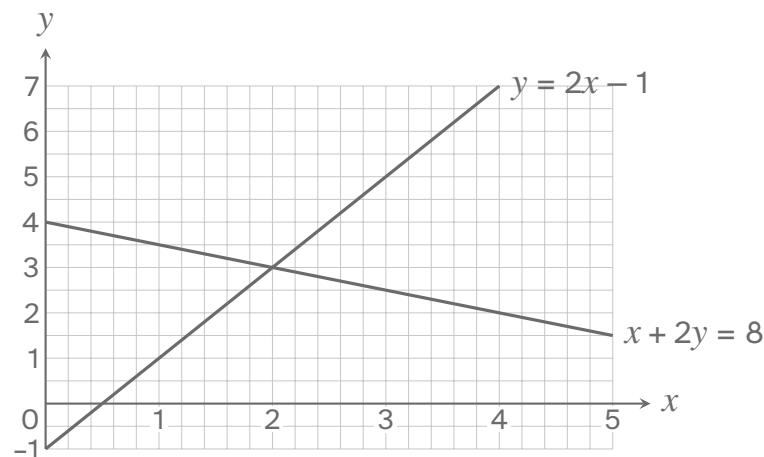
**Answer:**  $x \approx -0.7$  and  $x \approx 2.7$

**Tip:** The curve is symmetrical about  $x = 1$ , so the two solutions should be the same distance either side of 1. This is a good check.

*Marks:* 1 mark for using the line  $y = 2$ . 1 mark for both solutions.

**2** (2 marks)

The graphs of  $y = 2x - 1$  and  $x + 2y = 8$  are drawn on the grid.



Use the graphs to solve the simultaneous equations

$$y = 2x - 1$$

$$x + 2y = 8$$

The solution is the point that is on both lines.

That is where the two lines cross: (2, 3).

So  $x = 2$  and  $y = 3$ .

Check in both equations:

$$2 \times 2 - 1 = 3 \quad \checkmark$$

$$2 + 2 \times 3 = 8 \quad \checkmark$$

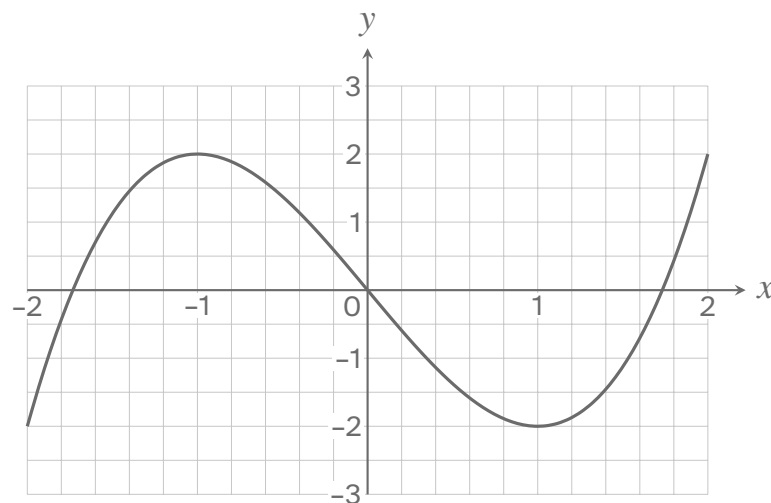
**Answer:**  $x = 2$ ,  $y = 3$

**Tip:** The question says use the graphs, so read the crossing point. Take care not to swap  $x$  and  $y$ .

*Marks:* 2 marks for both values correct. 1 mark for one correct value.

**3** (4 marks)

Here is the graph of  $y = x^3 - 3x$



- (a) Use the graph to estimate the three solutions of  $x^3 - 3x = 0$

The solutions are where  $y = 0$ , which is where the curve crosses the  $x$ -axis.

The curve crosses it three times:

$$x \approx -1.7, x = 0 \text{ and } x \approx 1.7$$

**Answer:**  $-1.7, 0, 1.7$

*Marks:* 2 marks for all three. 1 mark for two correct.

- (b) How many solutions does  $x^3 - 3x = 1$  have?

Give a reason for your answer.

The solutions are where the curve meets the line  $y = 1$ .

Draw the horizontal line  $y = 1$  on the graph.

It crosses the curve 3 times.

So there are 3 solutions.

**Answer:** 3, because the line  $y = 1$  crosses the curve three times

**Tip:** Count the crossing points with the line  $y = 1$ , not with the  $x$ -axis.

*Marks:* 1 mark for 3. 1 mark for the reason.

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**4** (1 mark)

Circle the equation whose solutions are where the graph of  $y = x^2 - 4$  meets the line  $y = 5$

Options:  $x^2 - 4 = 0$ ,  $x^2 = 5$ ,  $x^2 - 4 = 5$ ,  $x^2 + 1 = 0$

Where the graphs meet, they have the same  $y$ -value.

So  $x^2 - 4$  must equal 5.

The equation is  $x^2 - 4 = 5$ .

**Answer:**  $x^2 - 4 = 5$

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**5** (4 marks)

(a) On the grid, draw the graph of  $y = 3x - 4$  for values of  $x$  from 0 to 4

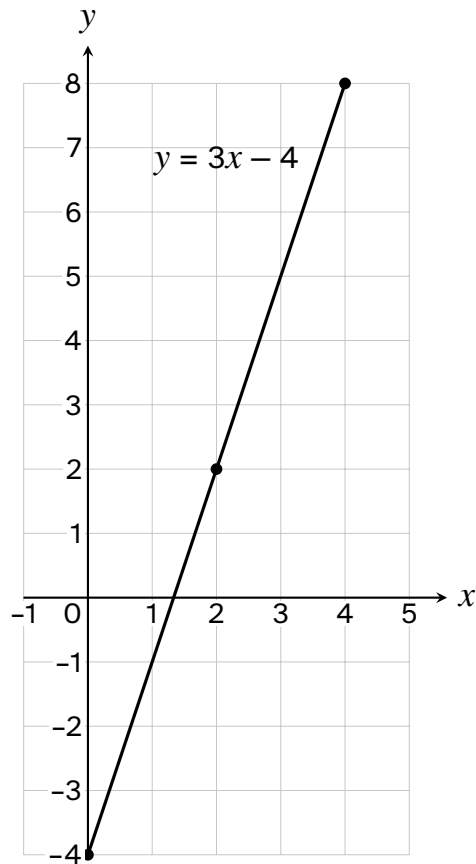
Work out some points.

$$x = 0: y = 3 \times 0 - 4 = -4$$

$$x = 2: y = 3 \times 2 - 4 = 2$$

$$x = 4: y = 3 \times 4 - 4 = 8$$

Plot the points and join them with a ruler, from  $x = 0$  to  $x = 4$ .



**Tip:** Work out at least three points. If they do not lie in a straight line, one of them is wrong.

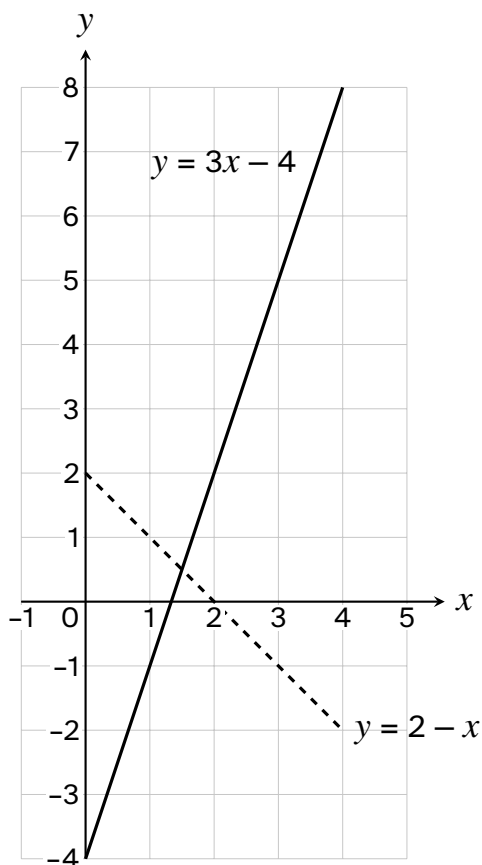
*Marks:* 2 marks for a correct line from  $x = 0$  to  $x = 4$ . 1 mark for at least 2 correct points plotted.

- (b) By drawing another straight line on the grid, solve  $3x - 4 = 2 - x$

The left side is the line you drew in part (a),  $y = 3x - 4$ .

The right side is a new line,  $y = 2 - x$ .

Draw  $y = 2 - x$ : it goes through (0, 2), (2, 0) and (4, -2).



The lines cross at (1.5, 0.5).

The answer is just the  $x$ -value:  $x = 1.5$

Check:  $3 \times 1.5 - 4 = 0.5$  and  $2 - 1.5 = 0.5$  ✓

**Answer:**  $x = 1.5$

**Tip:** Each side of the equation is a line. The solution is the  $x$ -value where the two lines cross, not the whole coordinate pair.

*Marks:* 1 mark for drawing the line  $y = 2 - x$ . 1 mark for  $x = 1.5$ .

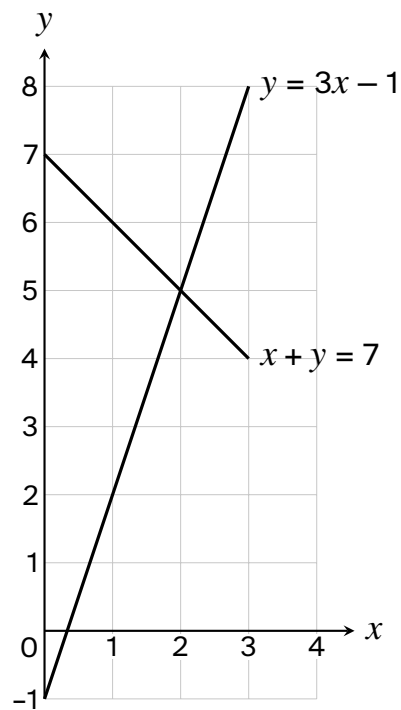
**6** (3 marks)

- (a) On the grid, draw the graphs of  $y = 3x - 1$  and  $x + y = 7$  for values of  $x$  from 0 to 3

$y = 3x - 1$ :  $x = 0$  gives  $y = -1$ ,  $x = 1$  gives  $y = 2$ ,  $x = 3$  gives  $y = 8$ .

$x + y = 7$ :  $x = 0$  gives  $y = 7$ ,  $x = 3$  gives  $y = 4$ .

Plot the points and join each set with a ruler.



Marks: 1 mark for each correct line.

- (b) Use your graphs to solve the simultaneous equations

$$y = 3x - 1$$

$$x + y = 7$$

The solution is where the two lines cross: (2, 5).

So  $x = 2$  and  $y = 5$ .

Check:  $3 \times 2 - 1 = 5$  and  $2 + 5 = 7$  ✓

**Answer:**  $x = 2$ ,  $y = 5$