

Sequences - Term-to-Term Rules and Patterns

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Write down the next number in each sequence.

(a) 5 9 13 17 ...

The terms go up by 4 each time.

$$17 + 4 = 21$$

Answer: 21

(b) 80 40 20 10 ...

Each term is half of the one before.

$$10 \div 2 = 5$$

Answer: 5

(c) 7 4 1 -2 ...

The terms go down by 3 each time.

$$-2 - 3 = -5$$

Answer: -5

Tip: The question asks for the next **number**, so write -5, not the rule “subtract 3”.

2 (1 mark)

Circle the term-to-term rule for the sequence 3 6 12 24 ...

Options: add 3, multiply by 2, add 6, multiply by 3

$$3 \times 2 = 6$$

$$6 \times 2 = 12$$

$$12 \times 2 = 24$$

Each term is double the one before.

Answer: multiply by 2

3 (3 marks)

Each term in a sequence is 3 times the term before it.

The 2nd term of the sequence is 4.5

Work out the 1st term and the 4th term.

To go forwards, multiply by 3.

$$\text{3rd term: } 4.5 \times 3 = 13.5$$

$$\text{4th term: } 13.5 \times 3 = 40.5$$

To go backwards, do the opposite of multiply by 3, which is divide by 3.

$$\text{1st term: } 4.5 \div 3 = 1.5$$

Check: 1.5, 4.5, 13.5, 40.5 ✓

Answer: 1st term 1.5, 4th term 40.5

Tip: From the 2nd term to the 4th term is 2 steps, so multiply by 3 twice. Going back to the 1st term is only 1 step.

Marks: 1 mark for $4.5 \div 3$ or 4.5×3 . 1 mark for 1.5. 1 mark for 40.5.

4 (3 marks)

The numbers in this sequence go up by the same amount each time.

7 _____ _____ _____ 29

Work out the three missing numbers.

From 7 to 29 there are 4 steps (gaps), not 5.

Total increase: $29 - 7 = 22$

Each step: $22 \div 4 = 5.5$

$7 + 5.5 = 12.5$, $12.5 + 5.5 = 18$, $18 + 5.5 = 23.5$

Check: $23.5 + 5.5 = 29$ ✓

Answer: 12.5, 18, 23.5

Tip: Count the gaps between terms, not the terms. There are 5 terms but only 4 gaps, so divide by 4, not 5 or 3.

Marks: 1 mark for $29 - 7 = 22$. 1 mark for $22 \div 4 = 5.5$. 1 mark for all three numbers correct.

5 (7 marks)

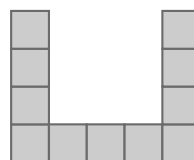
Here are some patterns made from tiles.



Pattern
number 1



Pattern
number 2



Pattern
number 3

(a) Draw pattern number 4 in the space below.

Each pattern has one more tile on the bottom row and one more tile on each side.

Pattern 4 has 6 tiles along the bottom and 4 tiles going up on each side.

That is $6 + 4 + 4 = 14$ tiles.

Answer: 14 tiles: 6 along the bottom and 4 up each side

- (b) Complete the table.

Pattern number	1	2	3	4	5
Number of tiles	5	8			

Count the tiles in pattern 3: $5 + 3 + 3 = 11$

Each pattern needs 3 more tiles than the one before (1 on the bottom and 1 on each side).

$$11 + 3 = 14$$

$$14 + 3 = 17$$

Answer: 11, 14, 17

Marks: 2 marks for all three correct. 1 mark for two correct.

- (c) How many tiles are needed for pattern number 12?

Pattern 5 needs 17 tiles.

From pattern 5 to pattern 12 is 7 more steps of +3.

$$17 + 7 \times 3 = 17 + 21 = 38$$

(Or: pattern n needs $3n + 2$ tiles, so $3 \times 12 + 2 = 38$.)

Answer: 38

Marks: 1 mark for continuing the pattern correctly or for $3 \times 12 + 2$. 1 mark for 38.

- (d) Bram has 100 tiles.
He makes the biggest pattern in this sequence that he can.
How many tiles does he have left over?

Pattern n uses $3n + 2$ tiles.

$$\text{Take off the 2 extra tiles: } 100 - 2 = 98$$

$$98 \div 3 = 32 \text{ remainder } 2$$

So the biggest pattern is pattern 32, which uses $3 \times 32 + 2 = 98$ tiles.

Tiles left over: $100 - 98 = 2$

Answer: 2

Tip: Check your pattern: pattern 33 would need $3 \times 33 + 2 = 101$ tiles, which is more than 100.

Marks: 1 mark for finding pattern 32 or 98 tiles. 1 mark for 2.

6 (2 marks)

Here are the first four terms of a sequence.

3 11 19 27

The 20th term of the sequence is 155

Work out the 23rd term.

The sequence goes up by 8 each time.

From the 20th term to the 23rd term is 3 steps.

$$155 + 3 \times 8 = 155 + 24 = 179$$

Answer: 179

Tip: You do not need to write out 23 terms. Just count the steps from the 20th term.

Marks: 1 mark for 3×8 (or 24). 1 mark for 179.

7 (1 mark)

The first four terms of a sequence are 6 10 14 18

Is 75 a term in this sequence?

Tick a box.

Give a reason for your answer.

The sequence starts at 6 and goes up by 4 each time.

Even + even is always even.

So every term is even.

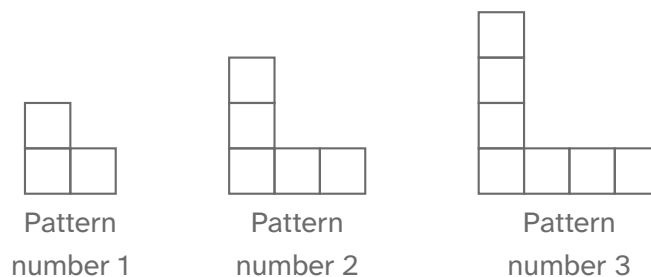
75 is odd, so it cannot be in the sequence.

Answer: No, because all the terms are even

Tip: Give a clear reason, e.g. “all the terms are even”, or show that 74 and 78 are terms. Just saying “it doesn’t reach 75” is not enough.

8 (2 marks)

Here are some patterns made from squares.



(a) How many squares are in pattern number 5?

Each pattern has 2 more squares than the one before.

3, 5, 7, 9, 11

Answer: 11

(b) Amy says,

‘Pattern number 20 has 40 squares.’

Is Amy correct? Tick a box.

Give a reason for your answer.

Pattern n has $2n + 1$ squares.

Pattern 20 has $2 \times 20 + 1 = 41$ squares.

(Also: the numbers of squares are all odd, and 40 is even.)

Answer: No, it has 41 squares

9 (3 marks)

The rule for a sequence is multiply by 2 then add 1

The 4th term of the sequence is 31

Work out the 1st term.

To go backwards, undo the rule: do the opposite operations in the reverse order.

The rule is multiply by 2 then add 1, so going backwards: subtract 1, then divide by 2.

3rd term: $31 - 1 = 30$, then $30 \div 2 = 15$

2nd term: $15 - 1 = 14$, then $14 \div 2 = 7$

1st term: $7 - 1 = 6$, then $6 \div 2 = 3$

Check: $3 \rightarrow 7 \rightarrow 15 \rightarrow 31$ ✓

Answer: 3

Tip: Reverse the **order** as well as the operations: subtract 1 first, then divide by 2. Doing $31 \div 2 - 1$ gives 14.5, which is wrong.

Marks: 1 mark for a correct step backwards, e.g. $(31 - 1) \div 2 = 15$. 1 mark for 7. 1 mark for 3.

10 (4 marks)

- (a) The 1st term of a sequence is 3.5
Each term after that is found using the rule

subtract 5 then multiply by 2

Work out the 2nd term and the 3rd term.

Do the rule in order: subtract 5 first, then multiply by 2.

2nd term: $3.5 - 5 = -1.5$, then $-1.5 \times 2 = -3$

3rd term: $-3 - 5 = -8$, then $-8 \times 2 = -16$

Answer: -3 and -16

Tip: Use **both** steps of the rule every time, in the right order. Take care with negatives: $-3 - 5 = -8$, not -2 .

Marks: 1 mark for -3. 1 mark for -16.

(b) Another sequence has the term-to-term rule

divide by 2 then subtract 3

The 1st term is 34

Work out the first term of this sequence that is a negative number.

Use the rule on each term until you get a negative number.

2nd term: $34 \div 2 = 17$, then $17 - 3 = 14$

3rd term: $14 \div 2 = 7$, then $7 - 3 = 4$

4th term: $4 \div 2 = 2$, then $2 - 3 = -1$

-1 is the first negative term (it is the 4th term).

Answer: -1

Tip: Divide first, then subtract, every time. Subtracting first ($34 - 3 = 31$, then $31 \div 2$) gives the wrong sequence.

Marks: 1 mark for 14 and 4. 1 mark for -1 .
